CSCI 1470/2470
Spring 2023

Ritambhara Singh

April 05, 2023
Wednesday

Deep Learning

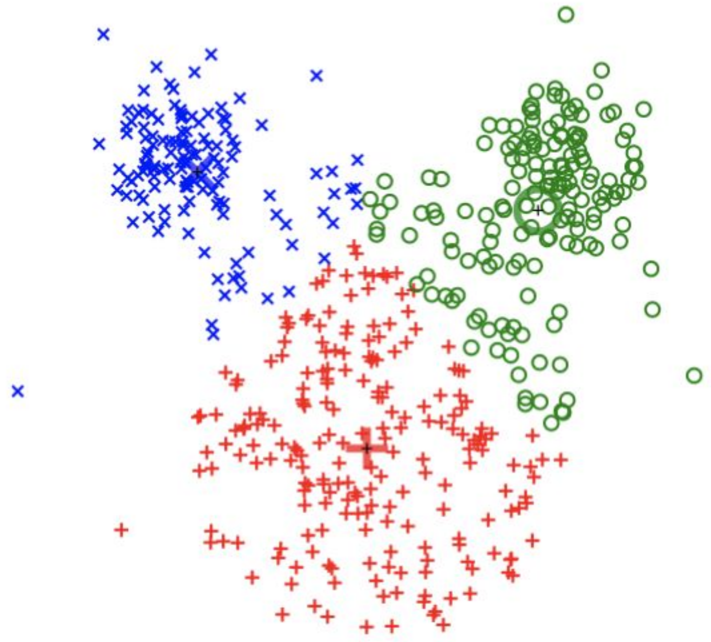
DALL-E 2 prompt “a painting of deep underwater with a yellow submarine in the bottom right corner”

Review: Supervised v/s Unsupervised Learning

Supervised Learning	**Unsupervised** Learning
Data: (x, y) x is data, y is label	**Data:** x Just data, no labels!
Goal: Learn a *function* to map x→y	**Goal:** Learn some underlying hidden *structure* of the data
Examples: Classification, regression, object detection, semantic segmentation, image captioning, *etc.*	**Examples:** Clustering, dimensionality reduction, feature learning, density estimation, *etc.*

https://www.cs.ubc.ca/~lsigal/532L/Lecture11.pdf

Review: Unsupervised Learning



k-means clustering

This image is CC0 public domain

Unsupervised Learning

Data: x
Just data, no labels!

Goal: Learn some underlying hidden *structure* of the data

Examples: Clustering, dimensionality reduction, feature learning, density estimation, *etc.*

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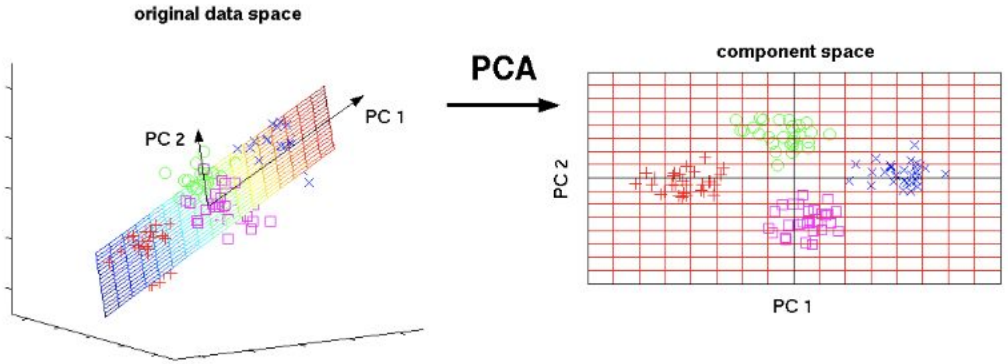
Review: Unsupervised Learning



dimensionality reduction

Unsupervised Learning

Data: x
Just data, no labels!

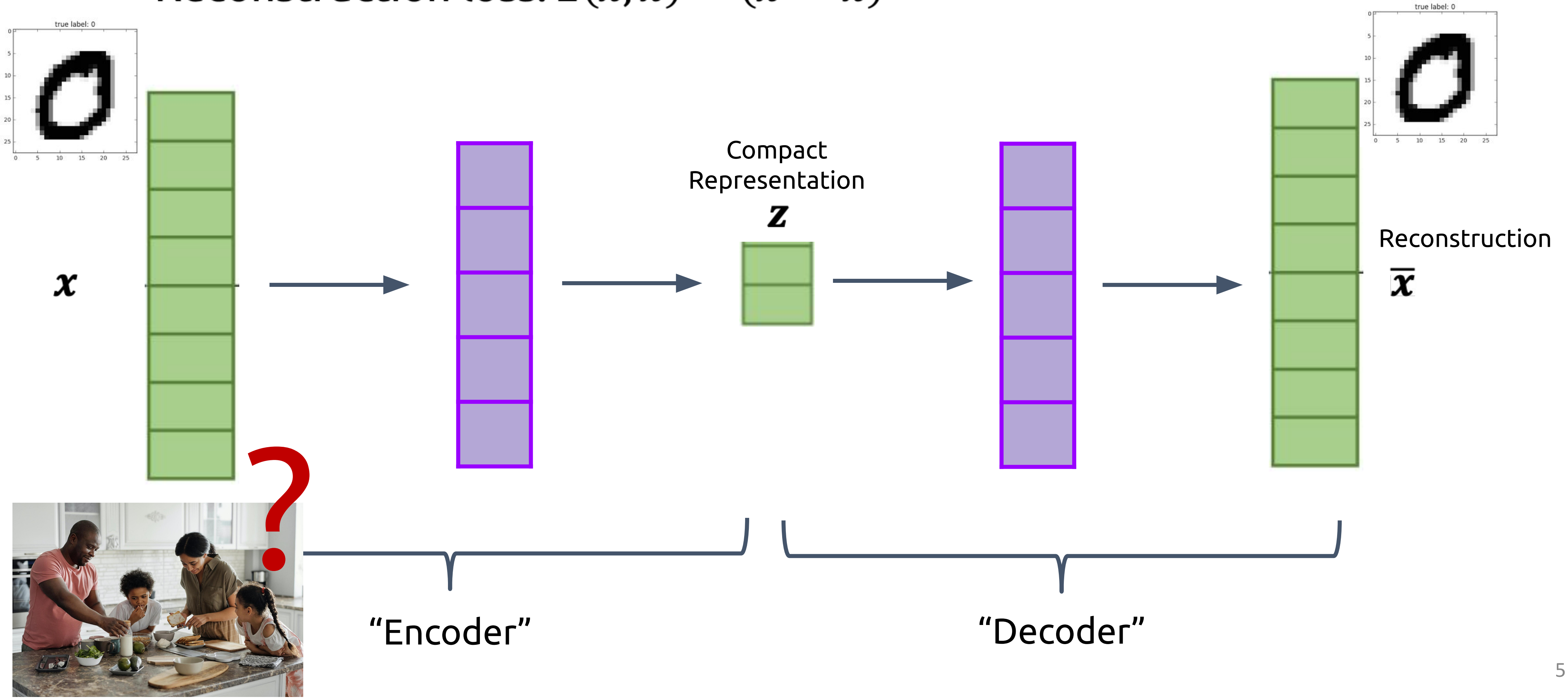
Goal: Learn some underlying hidden *structure* of the data

Examples: Clustering, dimensionality reduction, feature learning, density estimation, *etc.*

https://www.cs.ubc.ca/~lsigal/532L/Lecture11.pdf

Review: Autoencoder

- Reconstruction loss: $L(\boldsymbol{x}, \overline{\boldsymbol{x}}) = (\boldsymbol{x} - \overline{\boldsymbol{x}})^2$



Today’s goal – learn about variational autoencoders (VAEs)

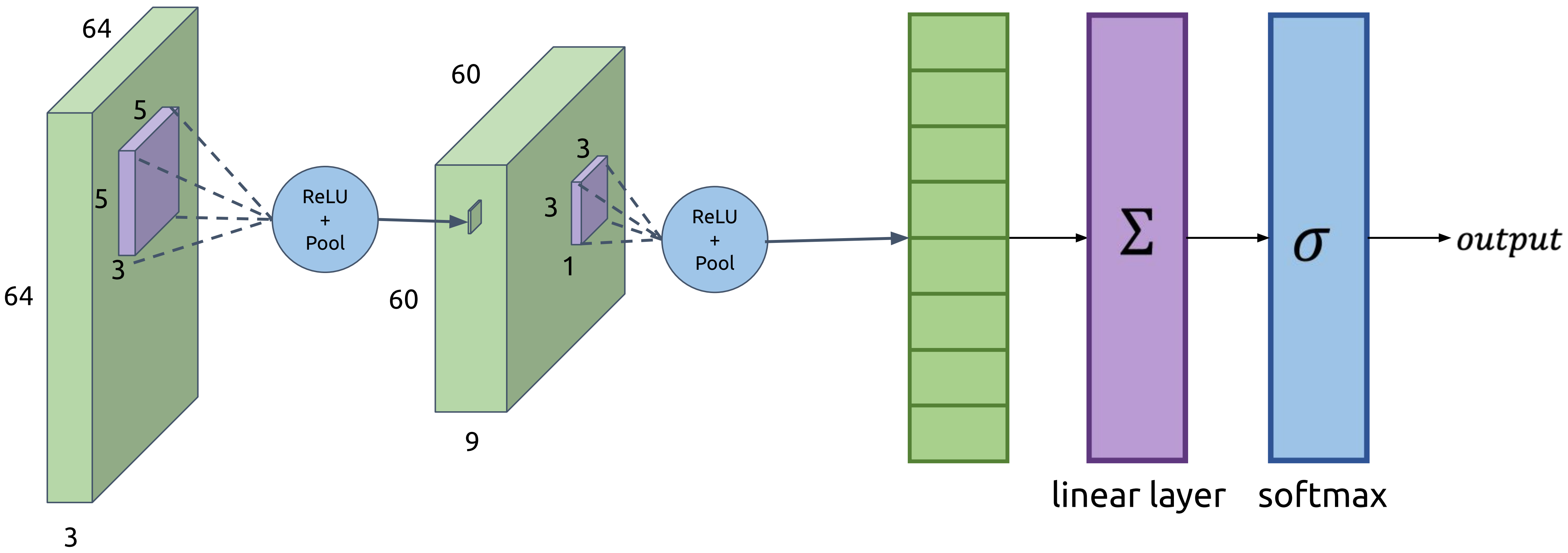
(1) Convolutional AEs

(2) Generative models

(3) Variational Autoencoders (VAEs)

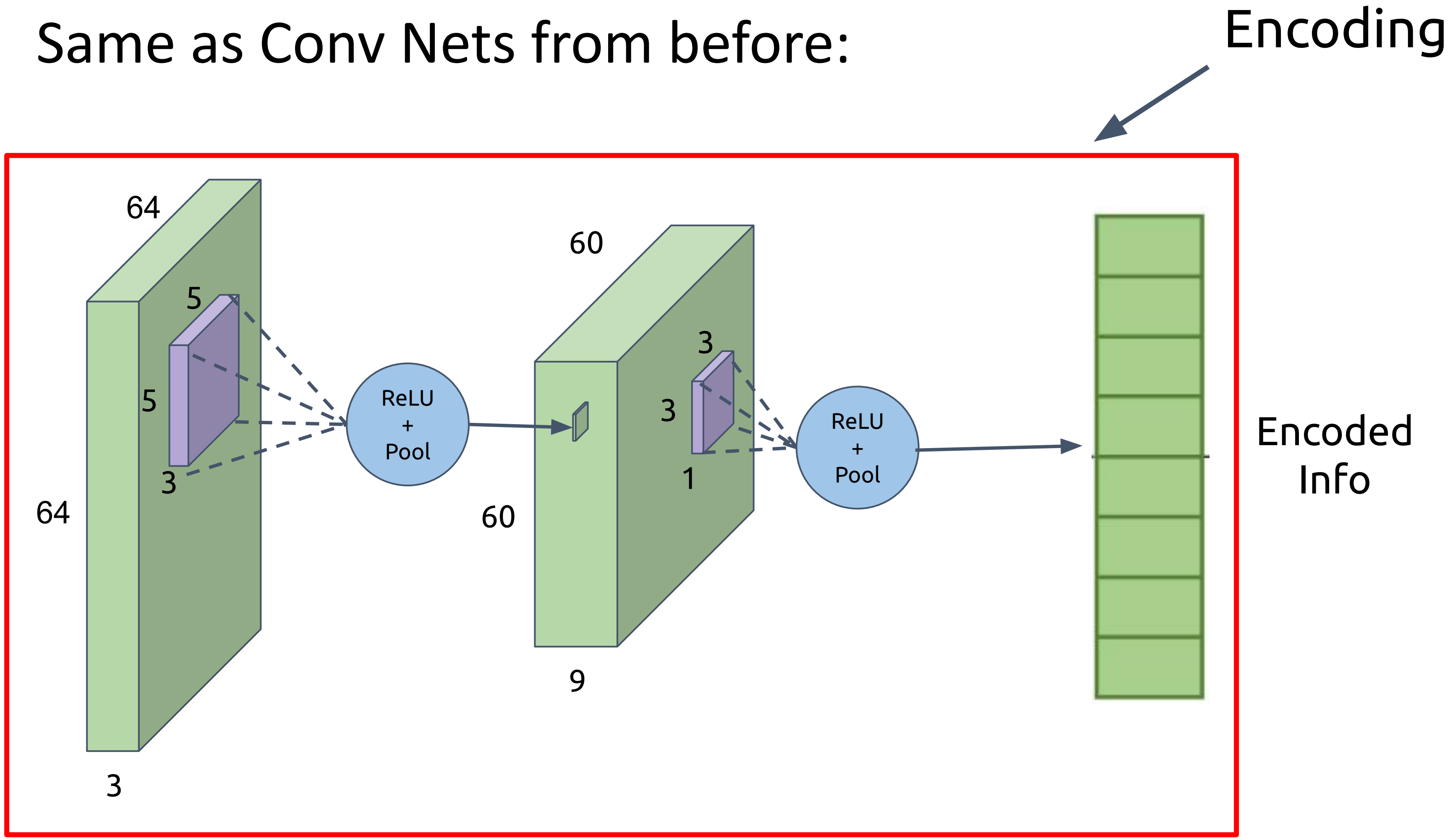
Convolutional Autoencoders

- CNNs are great for image processing in Neural Networks
- How can we build a ***convolutional*** autoencoder?



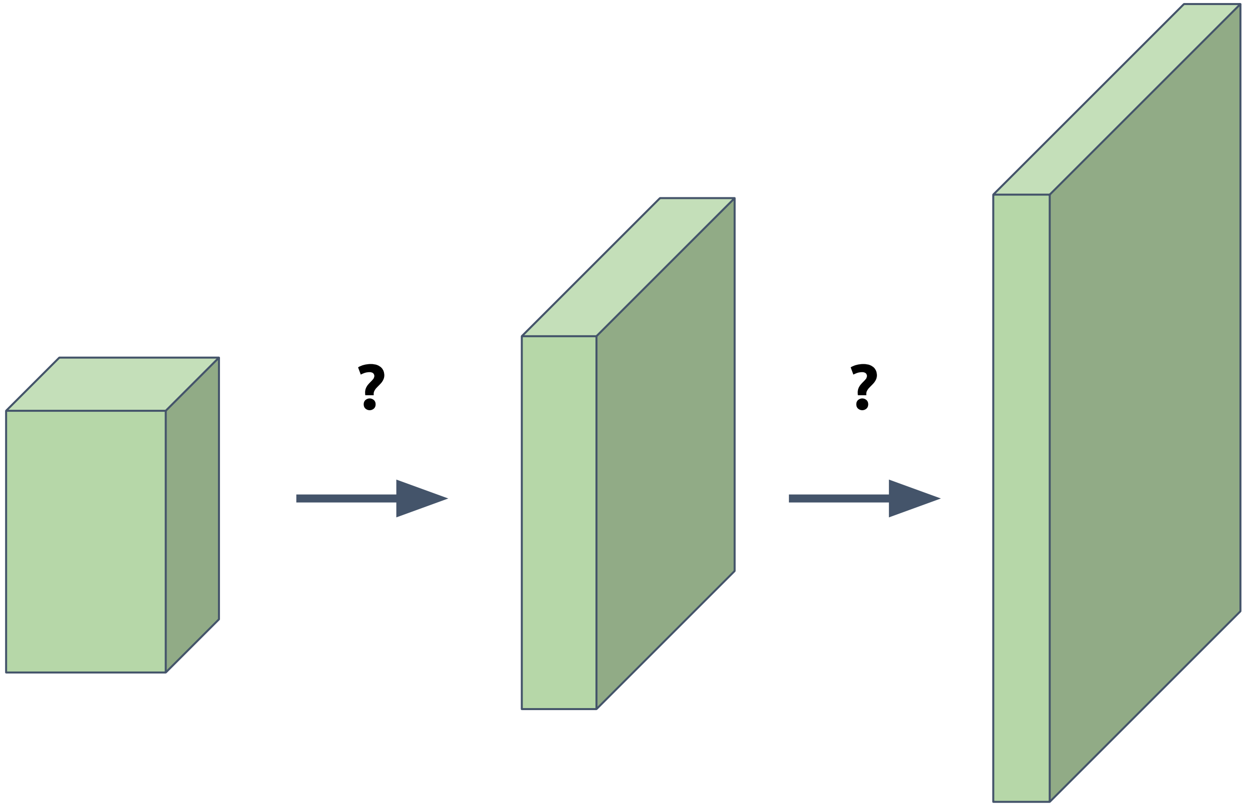
Convolutional Autoencoders: Encoding

Same as Conv Nets from before:



Autoencoders: Decoding

- Convolution as we know it only keeps resolution same or decreases it
- How do we go up in resolution?



Autoencoders: Transpose Convolution

- Convolution can be viewed as a matrix multiplication
- How do we represent it this way?

2	1	0	3
0	0	1	2
3	1	2	0
0	2	2	1

Input

?

1	2	3
4	5	6
7	8	9

Kernel

=

57	60
66	61

Output

Autoencoders: Transpose Convolution

Step 1: Flatten the image into a column vector

2	1	0	3
0	0	1	2
3	1	2	0
0	2	2	1

2
1
0
3
0
0
1
2
3
1
2
0
0
2
2
1

Autoencoders: Transpose Convolution

- Step 2: Unroll the kernel/filter

1	2	3
4	5	6
7	8	9

1	2	3	0	4	5	6	0	7	8	9	0	0	0	0	0
0	1	2	3	0	4	5	6	0	7	8	9	0	0	0	0
0	0	0	0	1	2	3	0	4	5	6	0	7	8	9	0
0	0	0	0	0	1	2	3	0	4	5	6	0	7	8	9

Autoencoders: Transpose Convolution

Step 3: Matrix multiply unrolled kernel with flattened image

$$
\begin{bmatrix}
1 & 2 & 3 & 0 & 4 & 5 & 6 & 0 & 7 & 8 & 9 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 2 & 3 & 0 & 4 & 5 & 6 & 0 & 7 & 8 & 9 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 2 & 3 & 0 & 4 & 5 & 6 & 0 & 7 & 8 & 9 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 2 & 3 & 0 & 4 & 5 & 6 & 0 & 7 & 8 & 9
\end{bmatrix}
\times
\begin{bmatrix} 2 \\ 1 \\ 0 \\ 3 \\ 0 \\ 0 \\ 1 \\ 2 \\ 3 \\ 1 \\ 2 \\ 0 \\ 0 \\ 2 \\ 2 \\ 1 \end{bmatrix}
=
\begin{bmatrix} 57 \\ 50 \\ 66 \\ 61 \end{bmatrix}
$$

Autoencoders: Transpose Convolution

Each row of the convolution matrix corresponds to a dot product between filter and image patch:

1	2	3	0	4	5	6	0	7	8	9	0	0	0	0	0
0	1	2	3	0	4	5	6	0	7	8	9	0	0	0	0
0	0	0	0	1	2	3	0	4	5	6	0	7	8	9	0
0	0	0	0	0	1	2	3	0	4	5	6	0	7	8	9

2
1
0
3
0
0
1
2
3
1
2
0
0
2
2
1

2	1	0	3
0	0	1	2
3	1	2	0
0	2	2	1

Autoencoders: Transpose Convolution

Each row of the convolution matrix corresponds to a dot product between filter and image patch:

$$
\begin{bmatrix}
1 & 2 & 3 & 0 & 4 & 5 & 6 & 0 & 7 & 8 & 9 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 2 & 3 & 0 & 4 & 5 & 6 & 0 & 7 & 8 & 9 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 2 & 3 & 0 & 4 & 5 & 6 & 0 & 7 & 8 & 9 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 2 & 3 & 0 & 4 & 5 & 6 & 0 & 7 & 8 & 9
\end{bmatrix}
\times
\begin{bmatrix} 2 \\ 1 \\ 0 \\ 3 \\ 0 \\ 0 \\ 1 \\ 2 \\ 3 \\ 1 \\ 2 \\ 0 \\ 0 \\ 2 \\ 2 \\ 1 \end{bmatrix}
\Longleftrightarrow
\begin{bmatrix}
2 & 1 & 0 & 3 \\
0 & 0 & 1 & 2 \\
3 & 1 & 2 & 0 \\
0 & 2 & 2 & 1
\end{bmatrix}
$$

Autoencoders: Transpose Convolution

Each row of the convolution matrix corresponds to a dot product between filter and image patch:

$$
\begin{bmatrix}
1 & 2 & 3 & 0 & 4 & 5 & 6 & 0 & 7 & 8 & 9 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 2 & 3 & 0 & 4 & 5 & 6 & 0 & 7 & 8 & 9 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 2 & 3 & 0 & 4 & 5 & 6 & 0 & 7 & 8 & 9 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 2 & 3 & 0 & 4 & 5 & 6 & 0 & 7 & 8 & 9
\end{bmatrix}
\times
\begin{bmatrix} 2 \\ 1 \\ 0 \\ 3 \\ 0 \\ 0 \\ 1 \\ 2 \\ 3 \\ 1 \\ 2 \\ 0 \\ 0 \\ 2 \\ 2 \\ 1 \end{bmatrix}
\Longleftrightarrow
\begin{bmatrix}
2 & 1 & 0 & 3 \\
0 & 0 & 1 & 2 \\
3 & 1 & 2 & 0 \\
0 & 2 & 2 & 1
\end{bmatrix}
$$

Autoencoders: Transpose Convolution

Each row of the convolution matrix corresponds to a dot product between filter and image patch:

1	2	3	0	4	5	6	0	7	8	9	0	0	0	0	0
0	1	2	3	0	4	5	6	0	7	8	9	0	0	0	0
0	0	0	0	1	2	3	0	4	5	6	0	7	8	9	0
0	0	0	0	0	1	2	3	0	4	5	6	0	7	8	9

×

2
1
0
3
0
0
1
2
3
1
2
0
0
2
2
1

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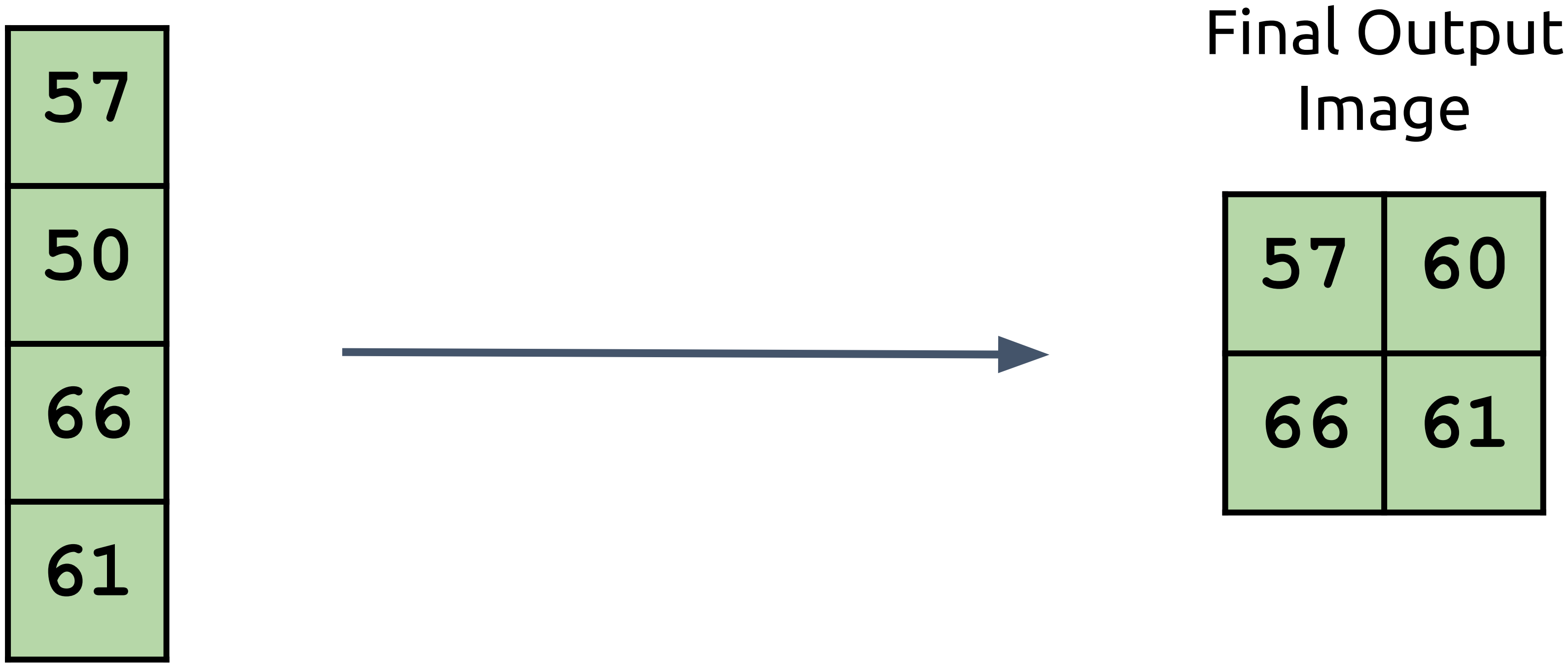
2	1	0	3
0	0	1	2
3	1	2	0
0	2	2	1

Autoencoders: Transpose Convolution

Step 4: Finally reshape the output back into a grid



Autoencoders: Transpose Convolution

$$
\begin{bmatrix}
1 & 2 & 3 & 0 & 4 & 5 & 6 & 0 & 7 & 8 & 9 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 2 & 3 & 0 & 4 & 5 & 6 & 0 & 7 & 8 & 9 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 2 & 3 & 0 & 4 & 5 & 6 & 0 & 7 & 8 & 9 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 2 & 3 & 0 & 4 & 5 & 6 & 0 & 7 & 8 & 9
\end{bmatrix}
\times
\begin{bmatrix} 2 \\ 1 \\ 0 \\ 3 \\ 0 \\ 0 \\ 1 \\ 2 \\ 3 \\ 1 \\ 2 \\ 0 \\ 0 \\ 2 \\ 2 \\ 1 \end{bmatrix}
=
\begin{bmatrix} 57 \\ 50 \\ 66 \\ 61 \end{bmatrix}
$$

Autoencoders: Transpose Convolution

To upsample an image, we just do the inverse of this operation.

What matrix do we use?

The **transpose** of the big convolution matrix

Input image flattened to column vector

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 2 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 4 & 0 & 1 & 0 \\ 5 & 4 & 2 & 1 \\ 6 & 5 & 3 & 2 \\ 0 & 6 & 0 & 3 \\ 7 & 0 & 4 & 0 \\ 8 & 7 & 5 & 4 \\ 9 & 8 & 6 & 5 \\ 0 & 9 & 0 & 6 \\ 0 & 0 & 7 & 0 \\ 0 & 0 & 8 & 7 \\ 0 & 0 & 9 & 8 \\ 0 & 0 & 0 & 9 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 0 \\ 6 \\ 10 \\ 14 \\ 3 \\ 15 \\ 22 \\ 26 \\ 6 \\ 14 \\ 23 \\ 26 \\ 9 \end{bmatrix}$$

Autoencoders: Transpose Convolution

Finally, reshape the output vector into a grid to get the final output image:

1
2
3
0
6
10
14
3
15
22
26
6
14
23
26
9

Final output image

1	2	3	4
6	10	14	3
15	1	22	26
14	23	26	9

Transpose Convolution in Tensorflow

```
tf.nn.conv2d_transpose(input, filters, output_shape, strides, padding='SAME')
```

4D tensor of shape [batch, height, width, in_channels]

4-D Tensor with shape [height, width, output_channels, in_channels]

length 4 1D tensor representing the output shape.

Strides along each dimension (list of integers)

String representing type of padding

Documentation here: https://www.tensorflow.org/versions/r2.0/api_docs/python/tf/nn/conv2d

Transpose Convolution in Tensorflow

```
tf.nn.conv2d_transpose(input, filters, output_shape, strides, padding='SAME')
```

Why do we need to specify output size?

Specifying Output Size

- An image can be the result of the same convolution on images of different resolution
- We need to specify which one we want.

57	60
66	61

1	2	3
4	5	6
7	8	9

Kernel

2	1	0	3
0	0	1	2
3	1	2	0
0	2	2	1

2	1	0	3	0
0	0	1	2	0
3	1	2	0	0
0	2	2	1	0
0	0	0	0	0

Any questions?

Transpose Convolution in Keras

```
tf.keras.layers.Conv2DTranspose(filters, kernel_size, strides, padding='SAME')
```

Number of filters (Integer)

Size of Convolution Window (tuple)

Strides along each dimension (list of integers)

String representing type of padding

Note: Output Shape is inferred, but can be specified via the "output_padding" parameter

Documentation here: https://www.tensorflow.org/api_docs/python/tf/keras/layers/Conv2DTranspose

Today's goal – learn about variational autoencoders (VAEs)

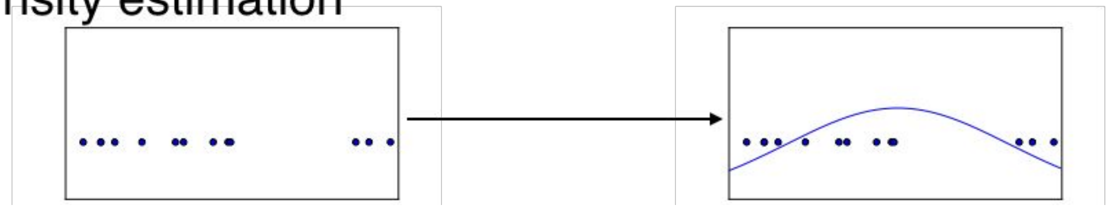
(1) Convolutional AEs

(2) Generative models

(3) Variational Autoencoders (VAEs)

Unsupervised Learning

- Density estimation



Unsupervised Learning

Data: x

Just data, no labels!

Goal: Learn some underlying hidden *structure* of the data

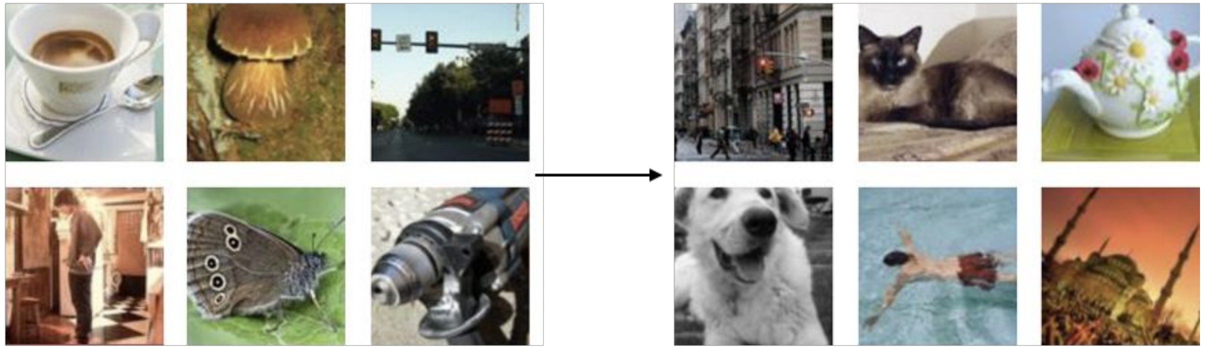
Examples: Clustering, dimensionality reduction, feature learning, $\boxed{\text{density estimation,}}$ *etc.*

Unsupervised Learning

Generative models

- Density estimation

- Sample generation



Training examples Model samples

Unsupervised Learning

Data: x
Just data, no labels!

Goal: Learn some underlying hidden *structure* of the data

Examples: Clustering, dimensionality reduction, feature learning, density estimation, *etc.*

https://www.cs.ubc.ca/~lsigal/532L/Lecture11.pdf

Discriminative v/s Generative models

Discriminative Model:
Learn a probability distribution p(y|x)

Generative Model:
Learn a probability distribution p(x)

Data: x

Label: y

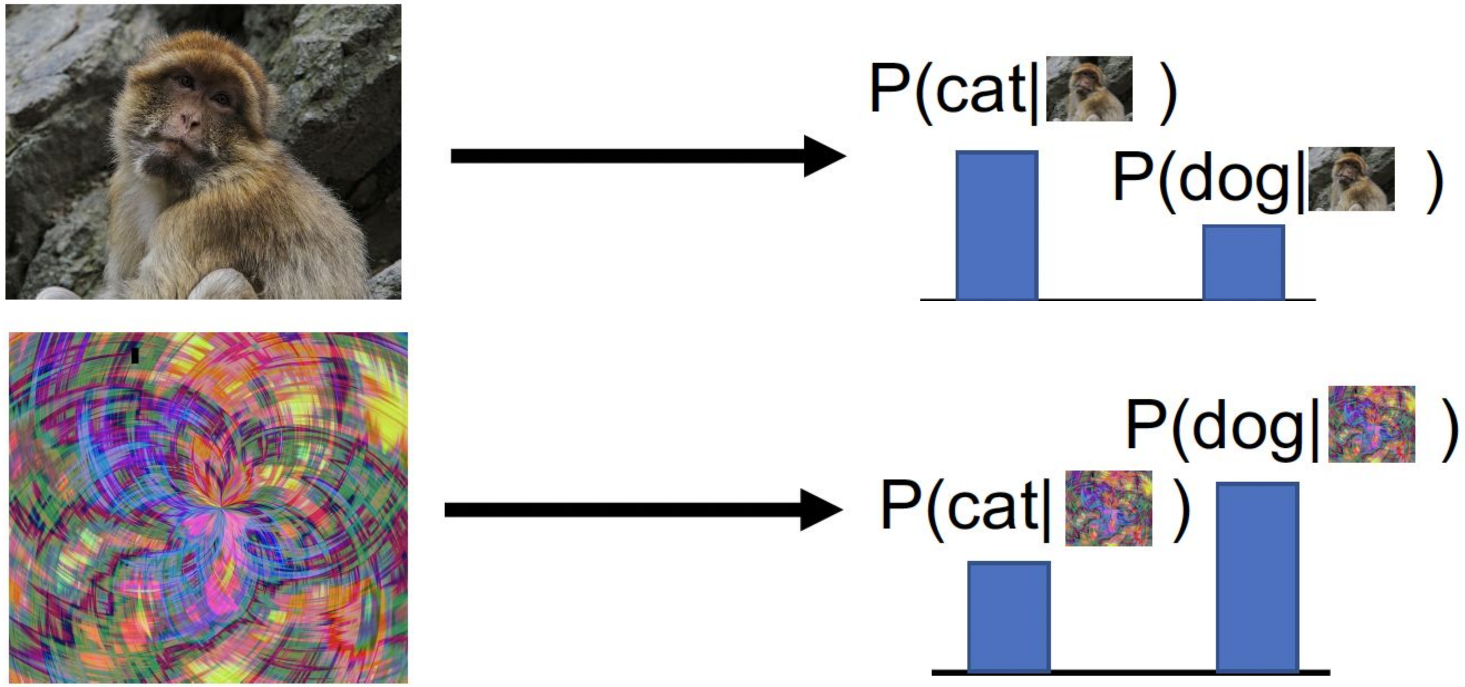
Cat

Credit: UMich EECS498

Discriminative v/s Generative models

Discriminative Model:
Learn a probability distribution p(y|x)

Generative Model:
Learn a probability distribution p(x)



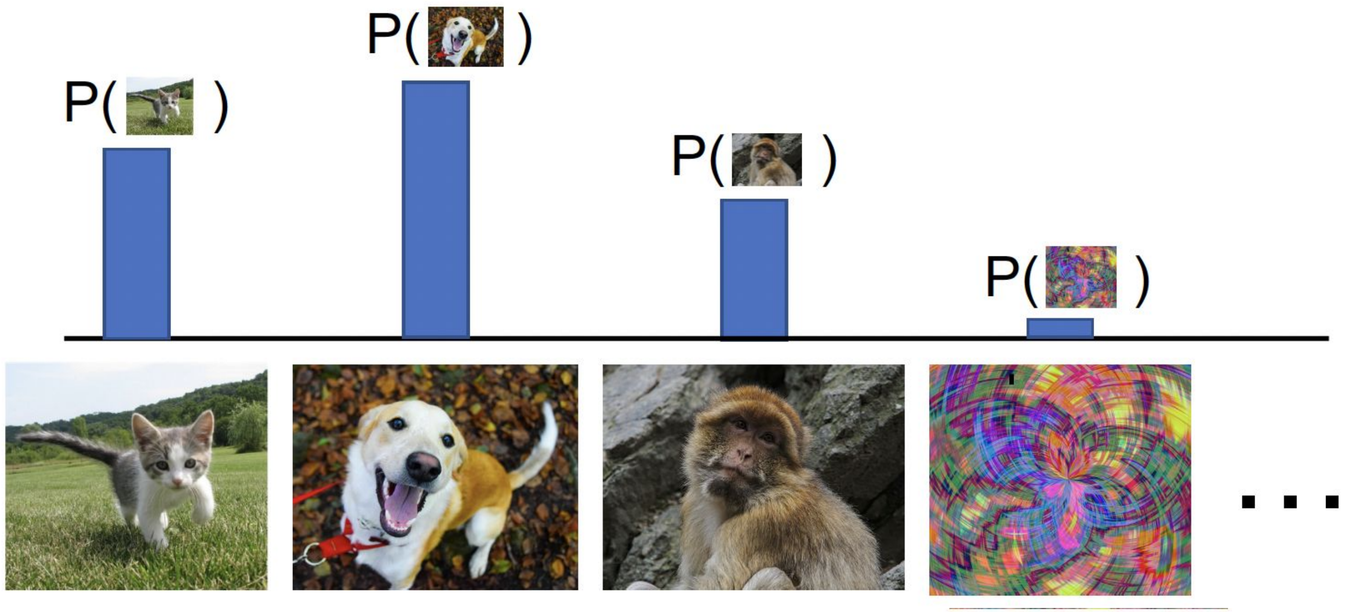
Discriminative model: No way for the model to handle unreasonable inputs; it must give label distributions for all images

Credit: UMich EECS498

Discriminative v/s Generative models

Discriminative Model:
Learn a probability distribution p(y|x)

Generative Model:
Learn a probability distribution p(x)

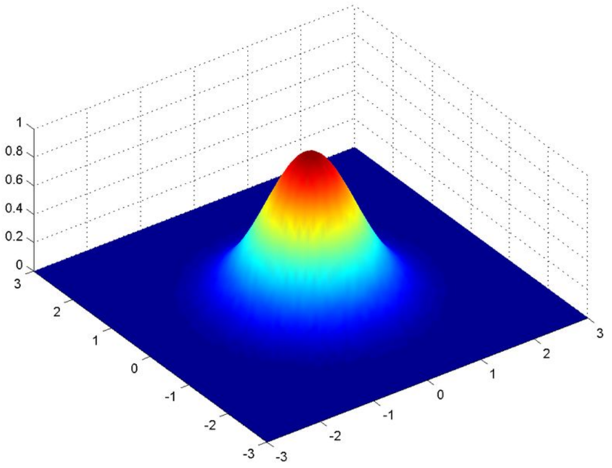


- Generative model: All possible images compete with each other for probability mass
- Intuition: Generation should require deep understanding! Is a dog more likely to sit or stand? How about 3-legged dog vs 3-armed monkey?
- Model can "reject" unreasonable inputs by assigning them small values

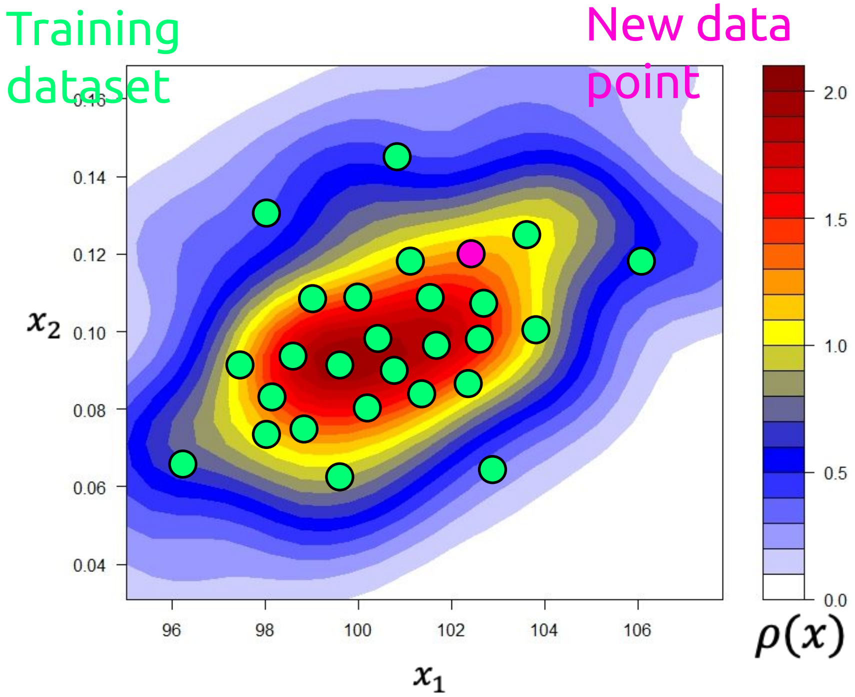
Credit: UMich EECS498

Generative Modeling Is:

1. A procedure for (approximately) ***sampling*** from the distribution from which a dataset was drawn



$$x' \sim p(.)$$

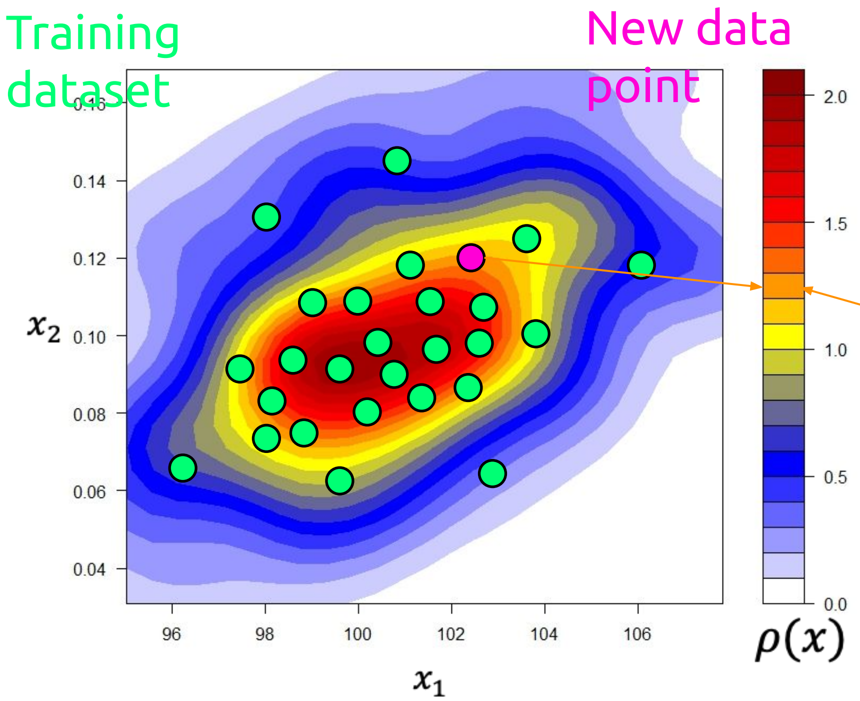


*Probability density is the relationship between observations and their probability.

Generative Modeling:

1. A procedure for (approximately) ***sampling*** from the distribution from which a dataset was drawn
2. A procedure for (approximately) ***evaluating the probability density*** of a datapoint under the distribution from which a dataset was drawn

$x' \sim p(.)$



$\rho(x')$

*Probability density is the relationship between observations and their probability.

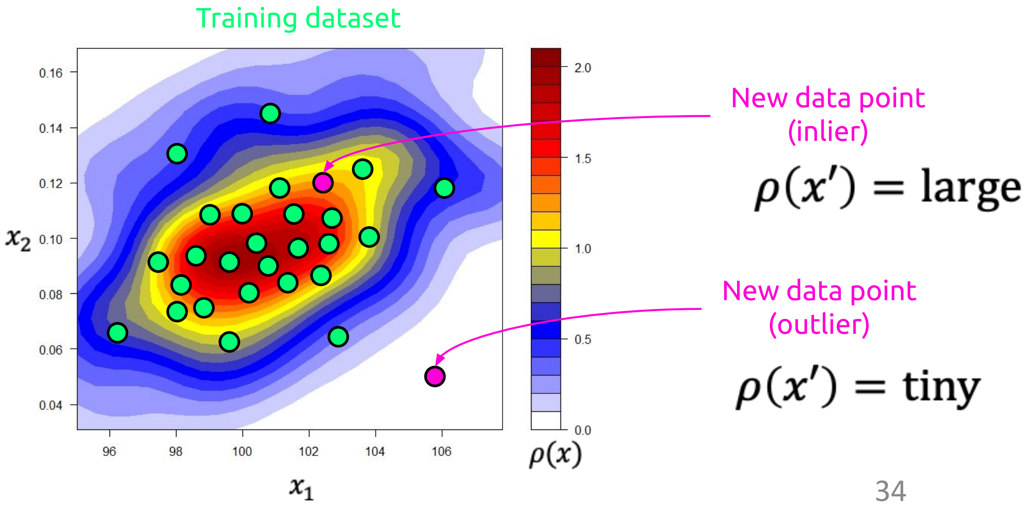
These two views are both useful

1. A procedure for (approximately) ***sampling*** from the distribution from which a dataset was drawn

2. A procedure for (approximately) ***evaluating the probability density*** of a datapoint under the distribution from which a dataset was drawn

Application: visual creativity

Application: outlier detection



These two views are both useful

1. A procedure for (approximately) ***sampling*** from the distribution from which a dataset was drawn

2. A procedure for (approximately) ***evaluating the probability density*** of a datapoint under the distribution from which a dataset was drawn

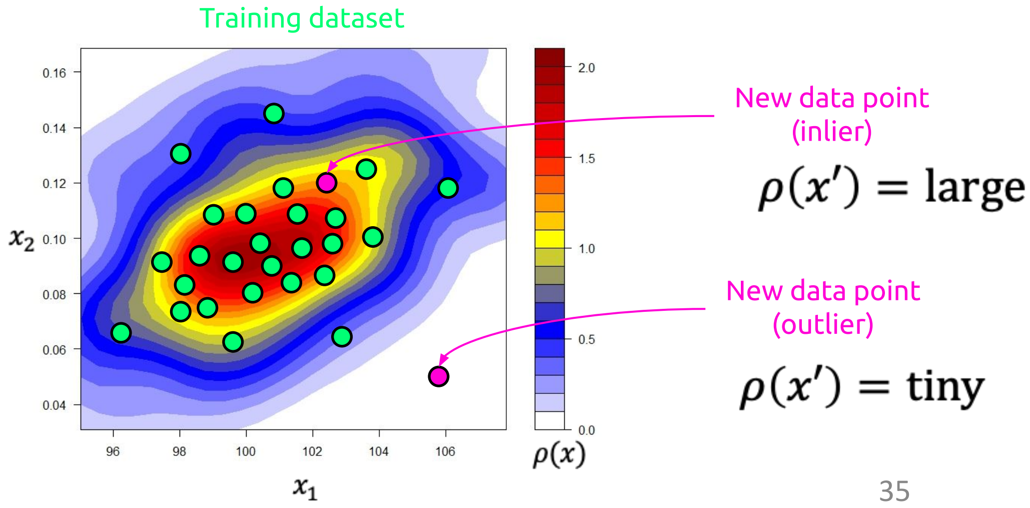
Application: things are getting more complicated!

Computer-generated inclusivity: fashion turns to 'diverse' AI models

Fashion brands including Levi's and Calvin Klein are having custom AI models created to 'supplement' representation in size, skin tone and age

The Guardian

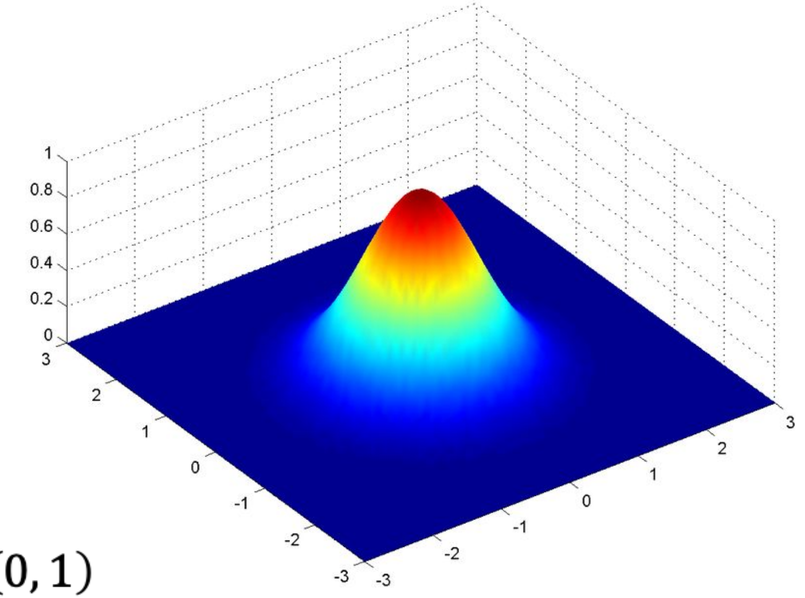
Application: outlier detection



What are some example generative models?

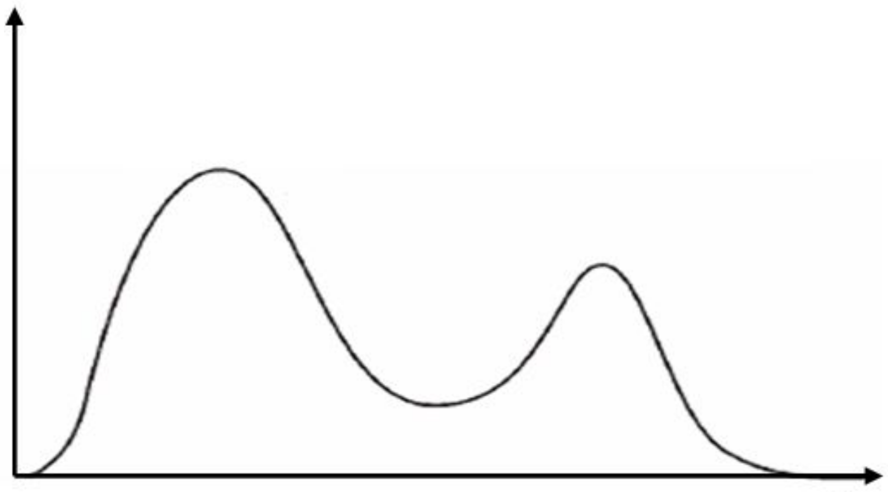
- Any probability distribution can be a generative model
- You already know some of these!

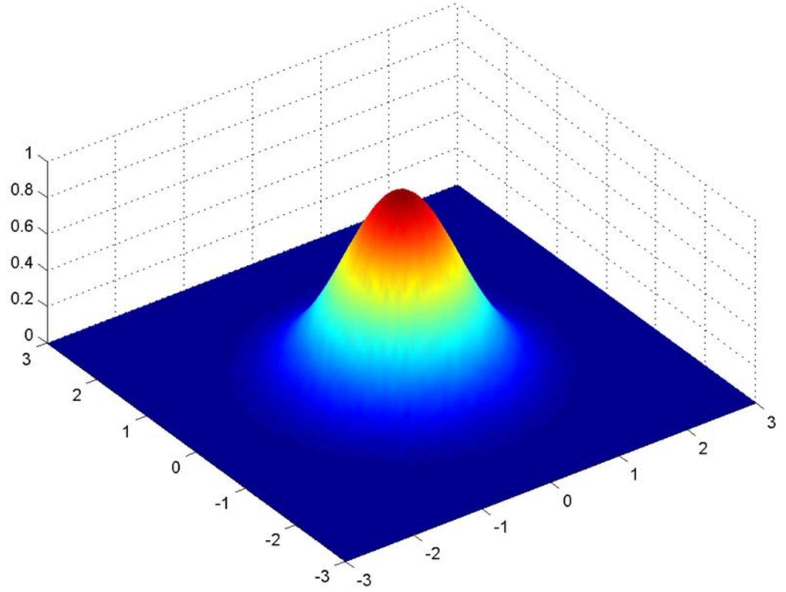
- E.g. The Gaussian Distribution
  - $p(x \mid \mu, \sigma) = \mathcal{N}(\mu, \sigma)(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$
  - Sampling:
    - Sample from the unit normal distribution $\rightarrow r \sim \mathcal{N}(0, 1)$
    - Return $\mu + r\sigma$



Disadvantages of Gaussian distribution

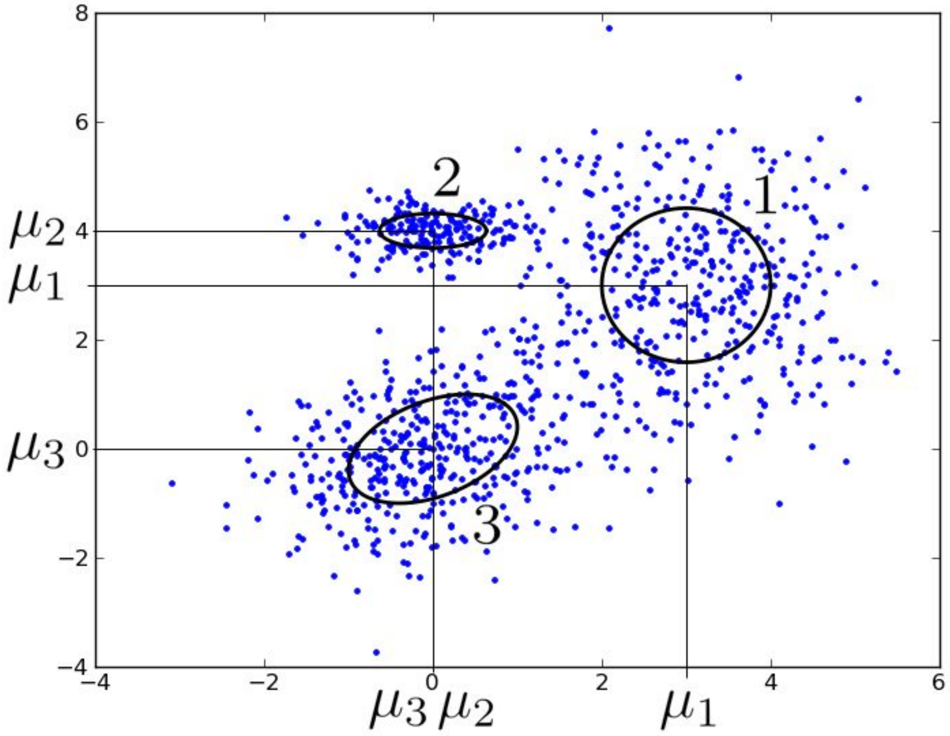
- Can only represent distributions with a single ***mode***
  - What if the distribution has multiple “peaks?”
  - E.g. book prices (concentrates around different price points if it’s hardcover, paperback, e-book, ...)



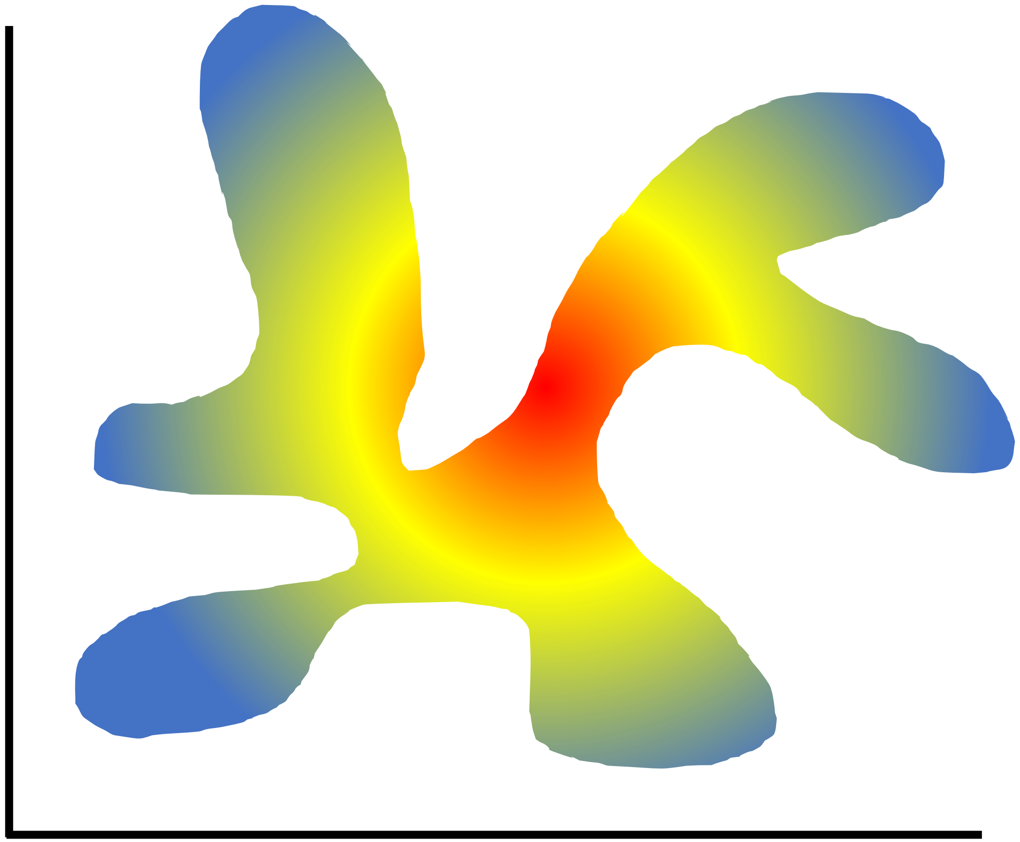


Better: ***Mixture*** of Gaussians

- A linear combination of multiple individual Gaussian distributions
  - $p(x \mid w, \mu, \sigma) = \sum_i w_i \mathcal{N}(\mu_i, \sigma_i)(x)$
  - Sampling:
    - Sample from the discrete weight distribution w to choose a Gaussian
    - Sample from that Gaussian as before



What about something like this?



- This doesn't look like a linear combination of Gaussians...
- ...but maybe it can be expressed as a ***nonlinear*** function of Gaussians?

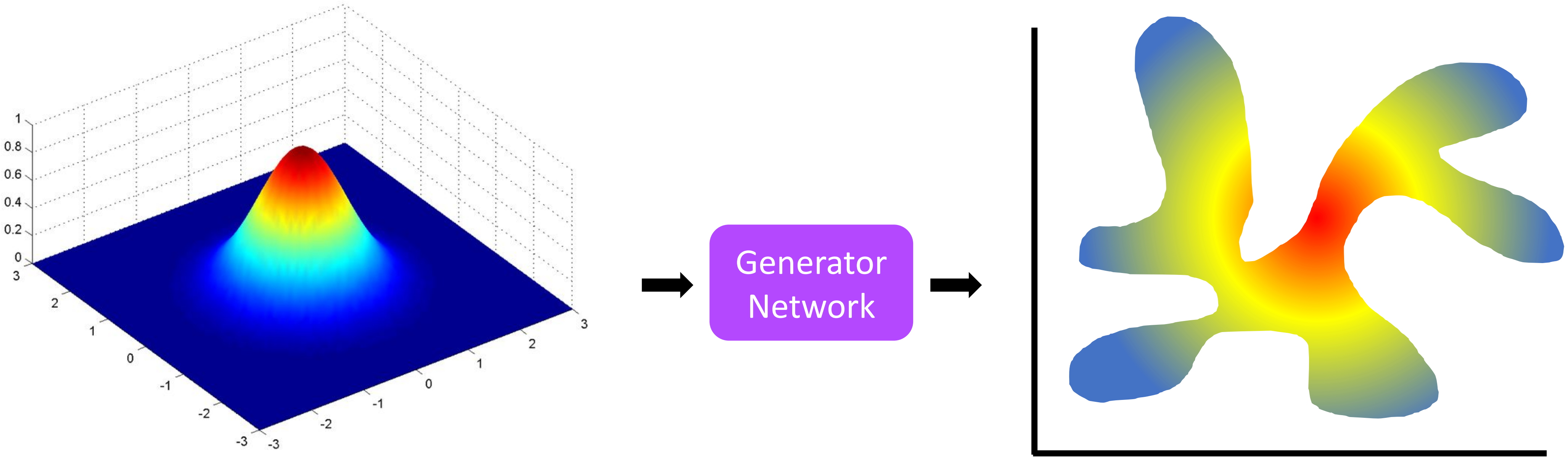
What can we do?

“I hear these neural nets are pretty good at learning non-linear functions” ☺️

Any questions?

A Neural Generative Model

- Input: a point $z \in \mathbb{R}^n$ drawn from a normal distribution $\mathcal{N}(\mu, \sigma)$
- Output: a point $x \in \mathbb{R}^m$ distributed according to some more complex distribution



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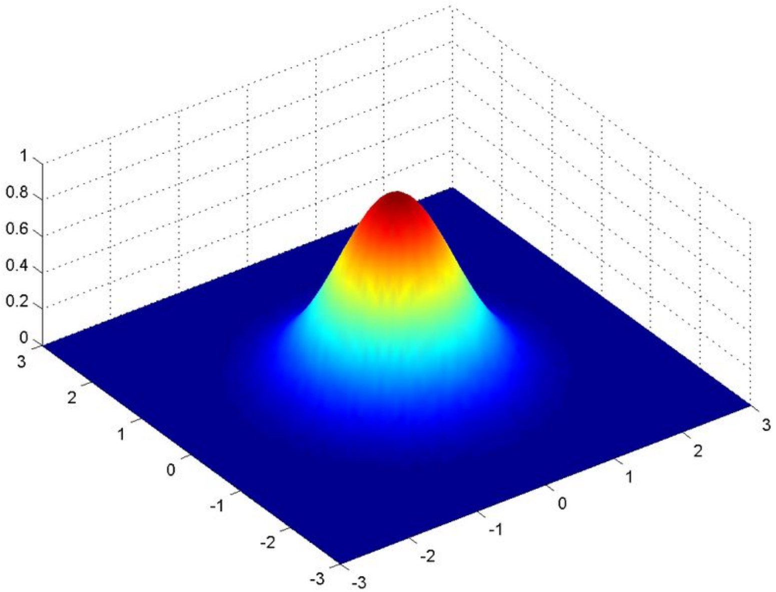
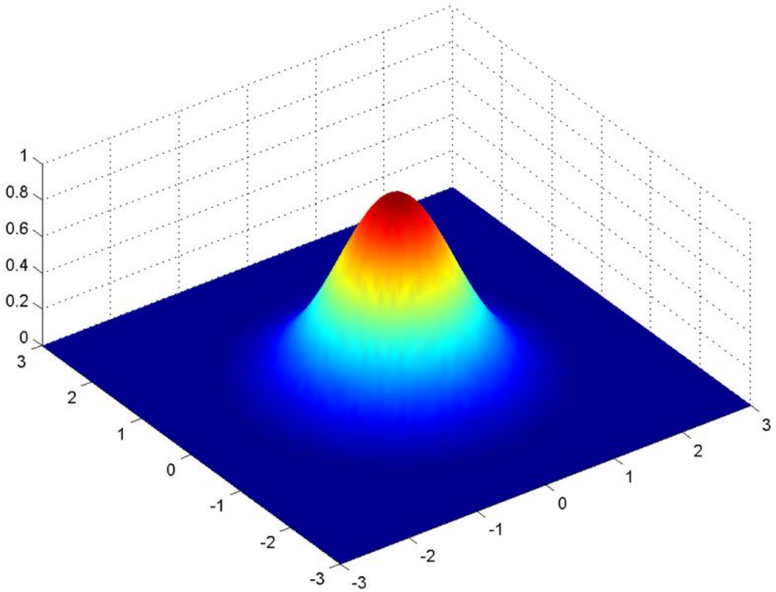


Generator Network

A Neural Generative Model

- Input: a point $z \in \mathbb{R}^n$ drawn from a normal distribution $\mathcal{N}(\mu, \sigma)$
- Output: a point $x \in \mathbb{R}^m$ distributed according to some more complex distribution

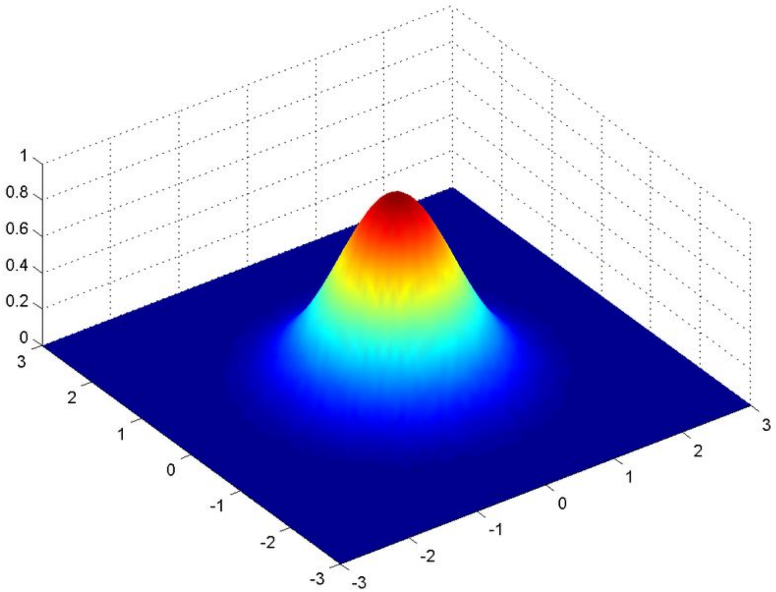


→ Generator Network →

The distribution of hu

A Neural Generative Model

- Great! So...how do we train this thing?
    - Let’s modify our autoencoder to achieve this

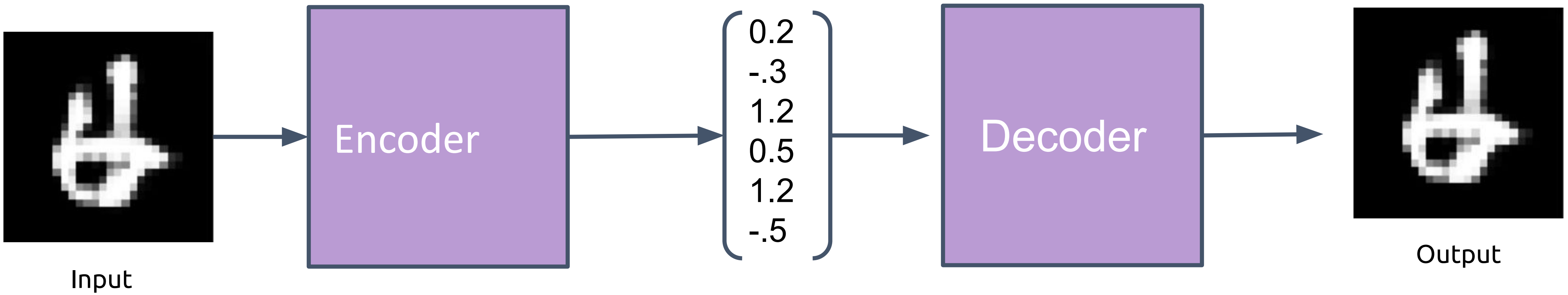


→ Generator Network →

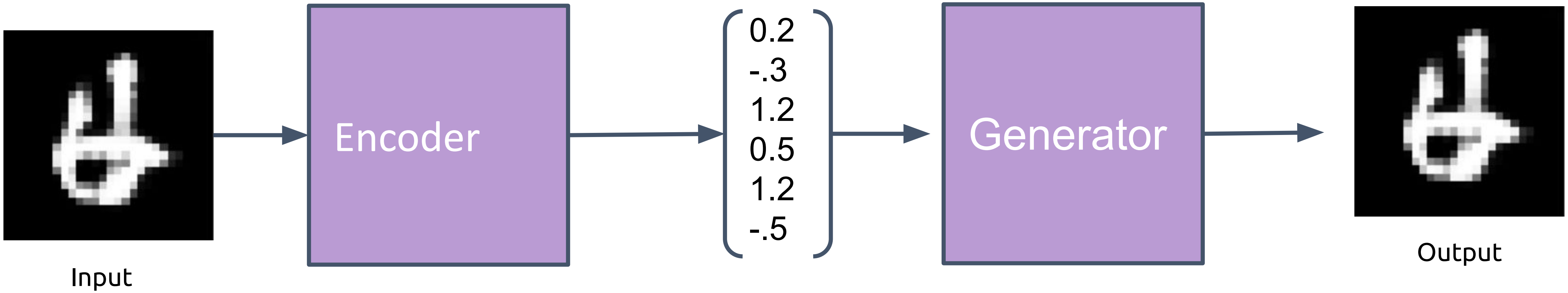
The distribution of
hu

Autoencoder

Let's think for a bit – how to modify the autoencoder to make it a generative model?

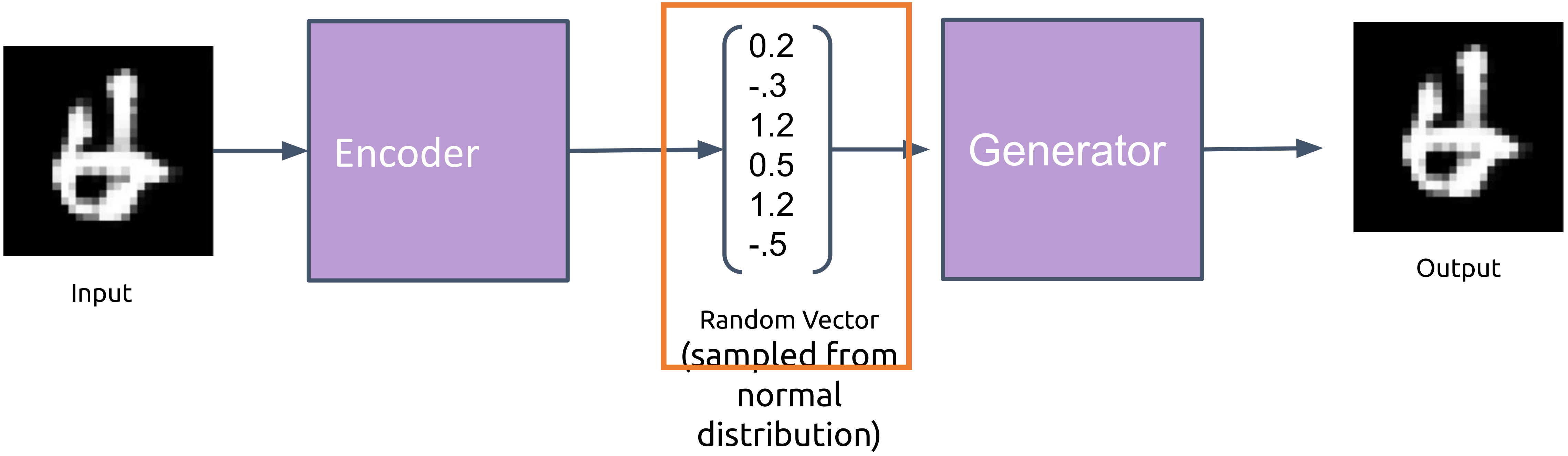


Variational Autoencoders (VAEs)



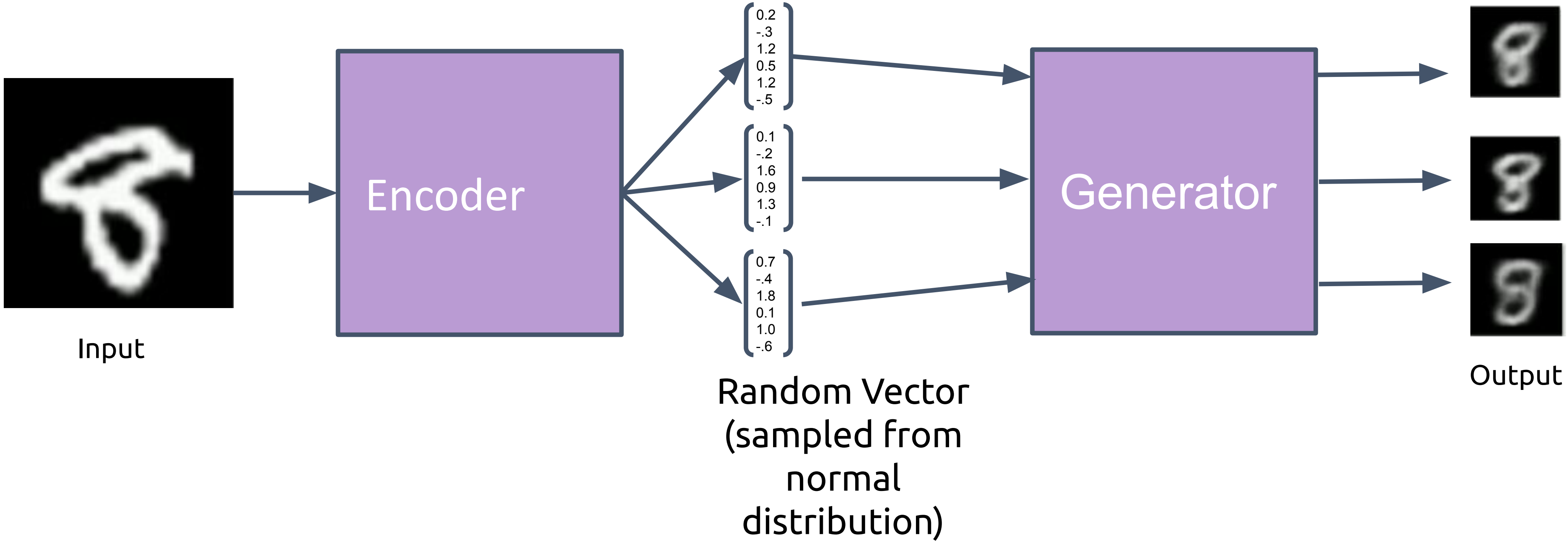
Variational Autoencoders (VAEs)

- This looks almost exactly like an autoencoder...
- ...except that this bottleneck vector is randomly sampled
  - We'll see how in a few slides



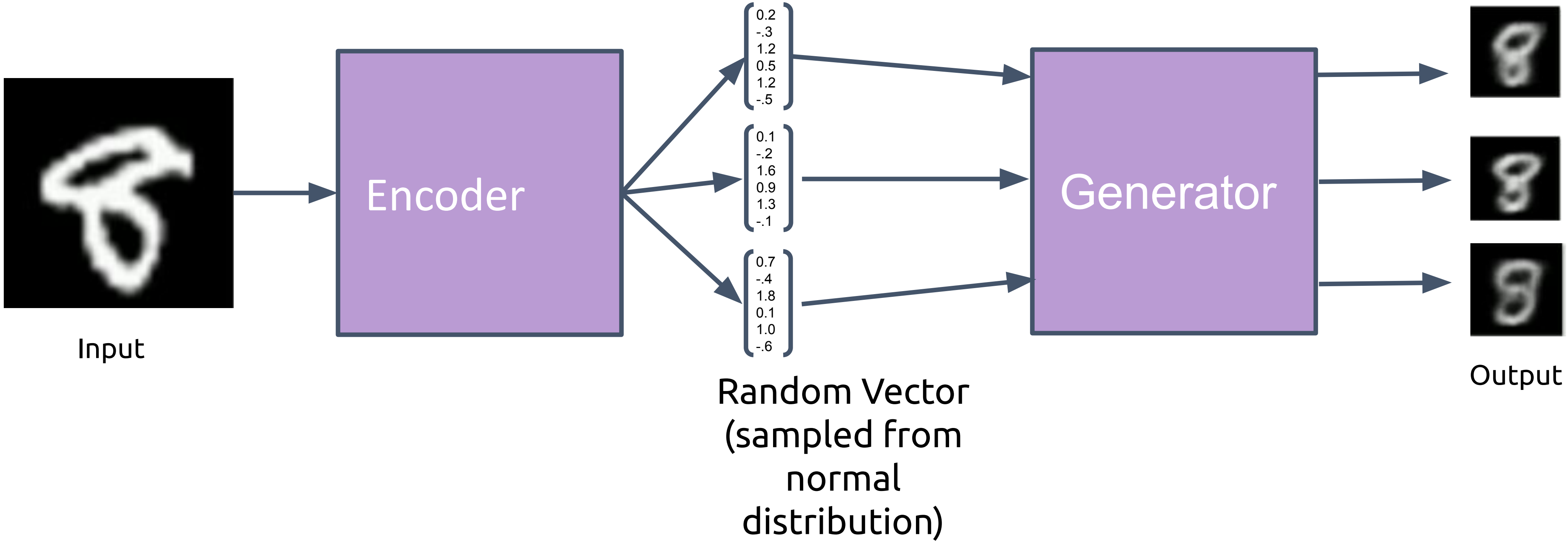
Variational Autoencoders (VAEs)

- In fact, the encoder can produce multiple different random vectors...
- ...which then lead to different outputs which are variants of the input



Variational Autoencoders (VAEs)

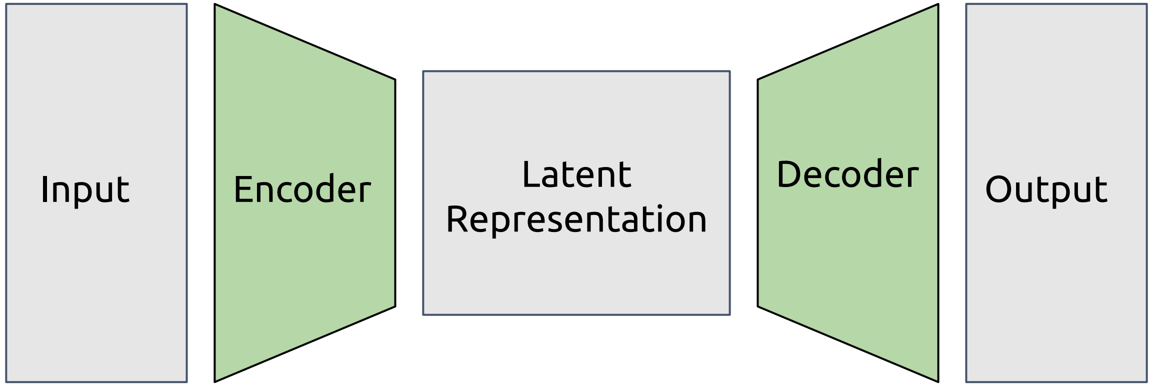
- ***Why do this?***
  - We'll see shortly how this setup allows for a nice, stable learning algorithm
  - (It's actually just a small modification to how autoencoders are trained)



Building up the VAE Architecture

If we were to describe an autoencoder functionally:

$$\texttt{Output = Decoder}(\underbrace{\texttt{Encoder(Input)}}_{\text{Latent Representation}})$$

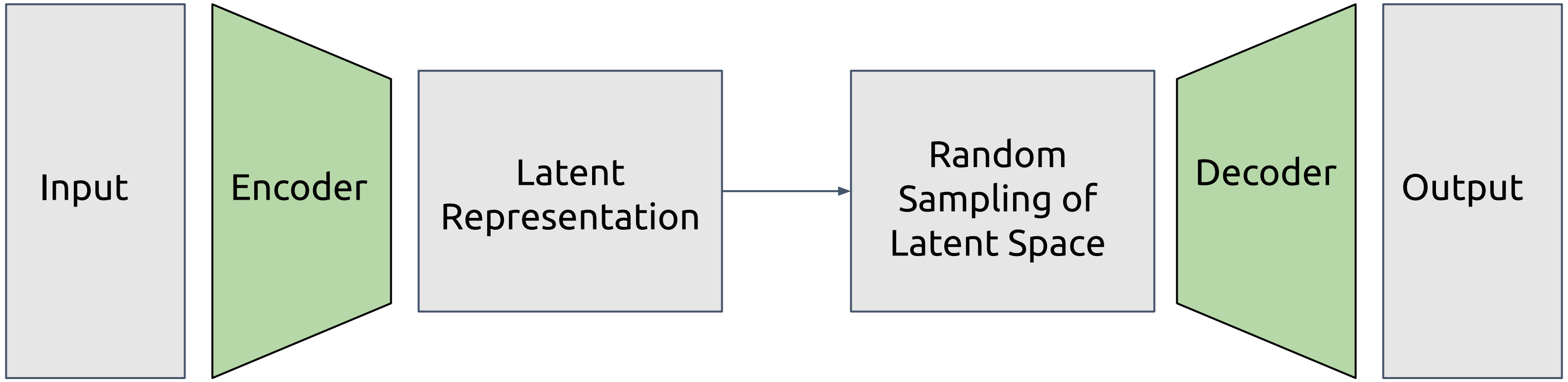


*Latent representation ~ compressed representation

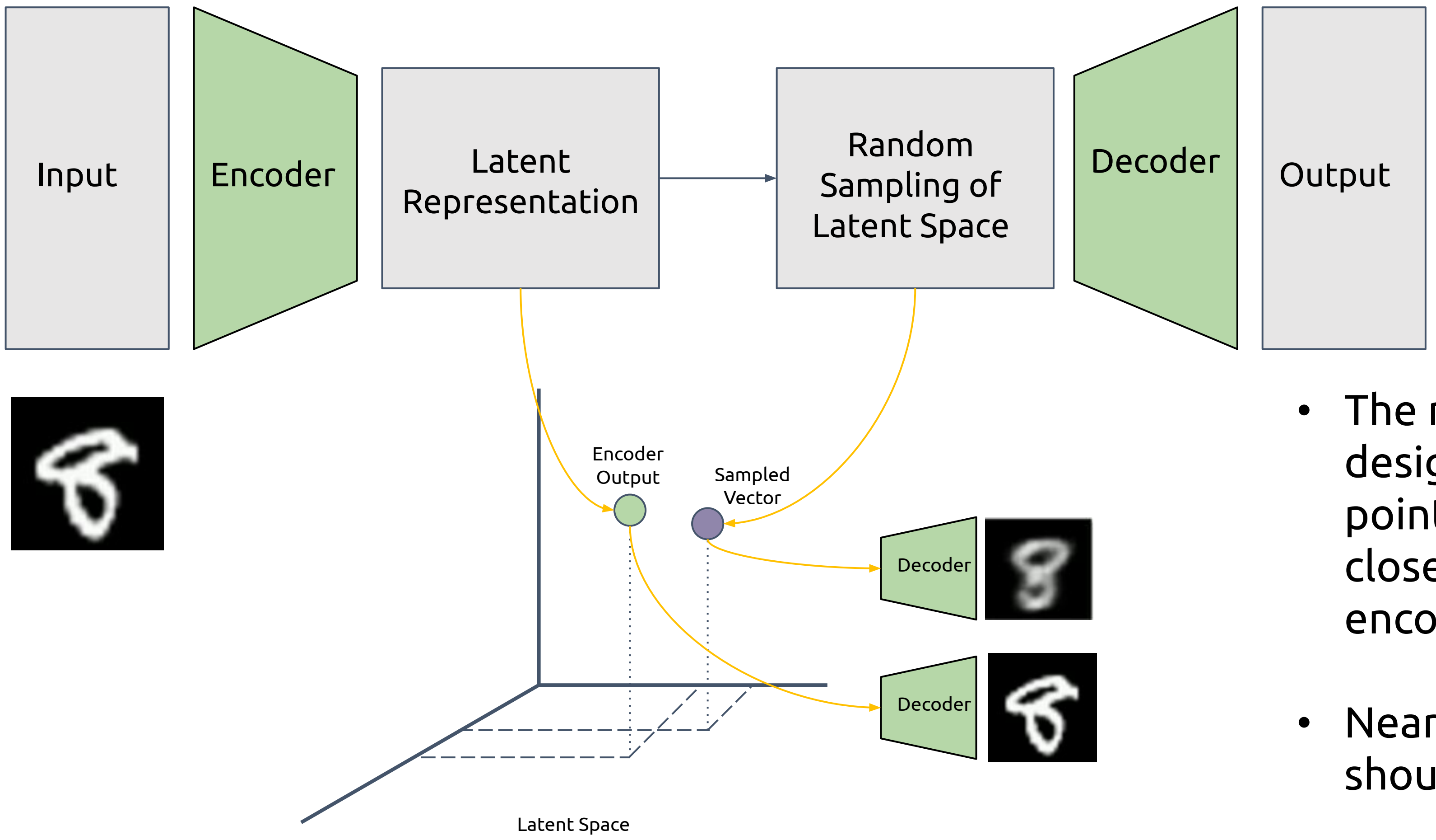
Building up the VAE Architecture

For variational autoencoders, we also do a random sampling operation at the bottleneck

```
Output = Decoder(random_sample(Encoder(Input)))
```



How does random sampling in latent space lead to variation?



- The random sampling should be designed to produce random points in latent space that are close to the output of the encoder

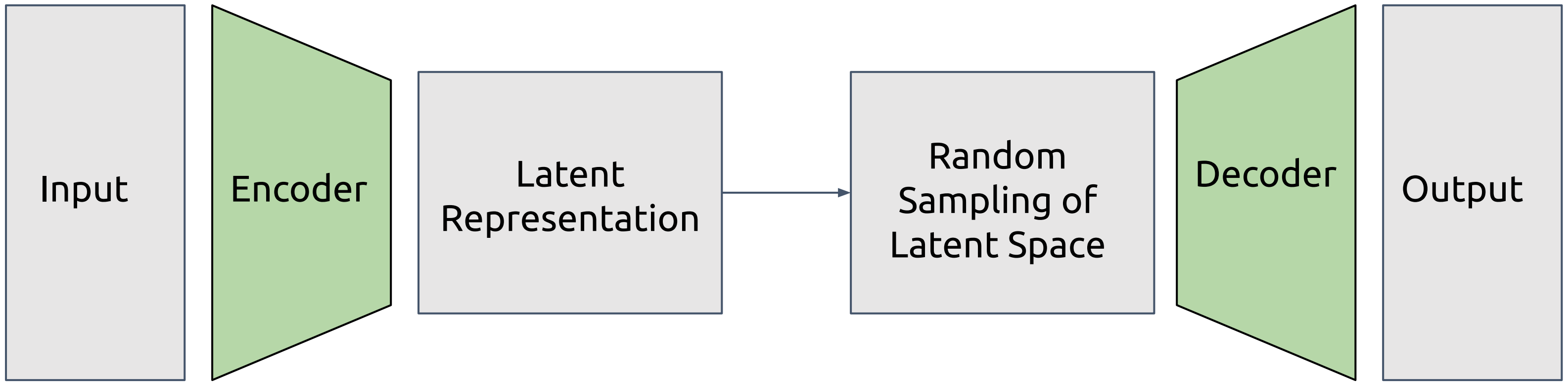
- Nearby points in the latent space should decode to similar images

How should `random_sample` be defined?

`Output = Decoder(`**`random_sample`**`(Encoder(Input)))`

- We want the sample to be close to the encoder output
- One option: sample from a Gaussian centered at `Encoder(Input)`

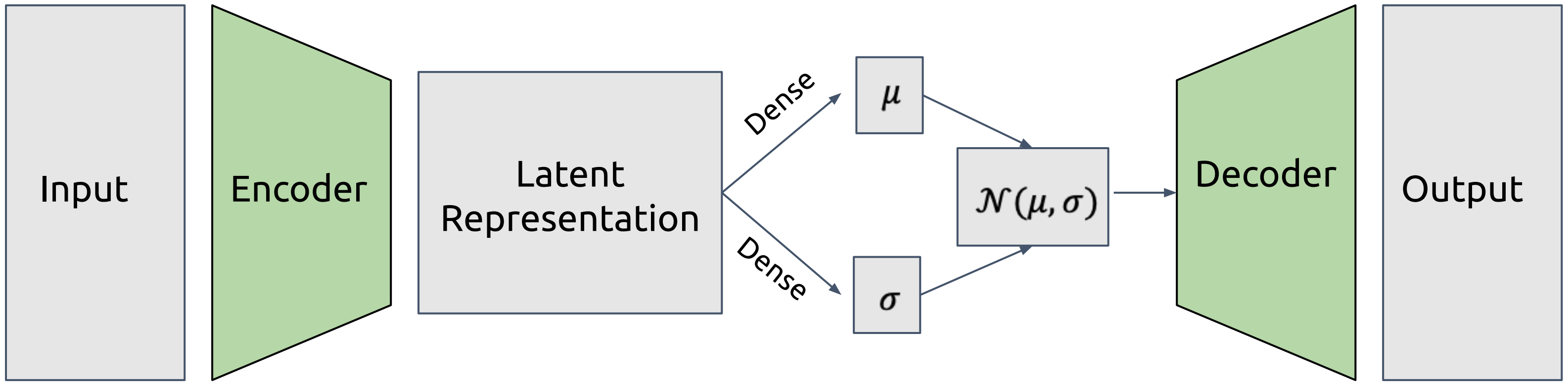
What can we modify?



How should `random_sample` be defined?

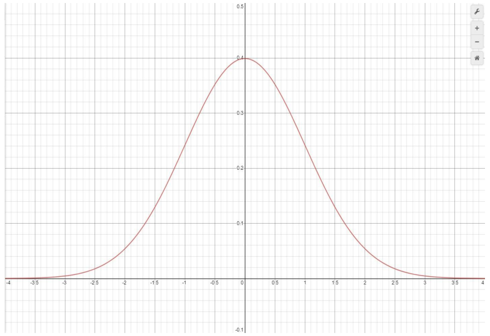
`Output = Decoder(random_sample(Encoder(Input)))`

- We want the sample to be close to the encoder output
- One option: sample from a Gaussian centered at `Encoder(Input)`
- Use two dense layers to convert the encoder output into the mean and standard deviation of the Gaussian

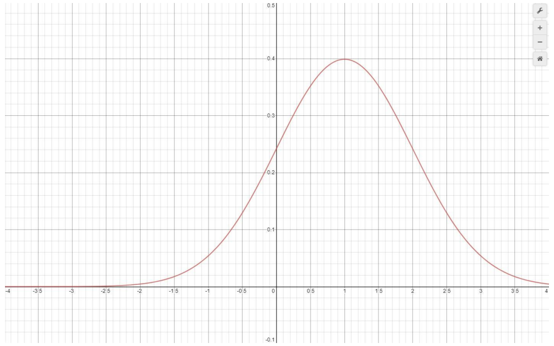


Any questions?

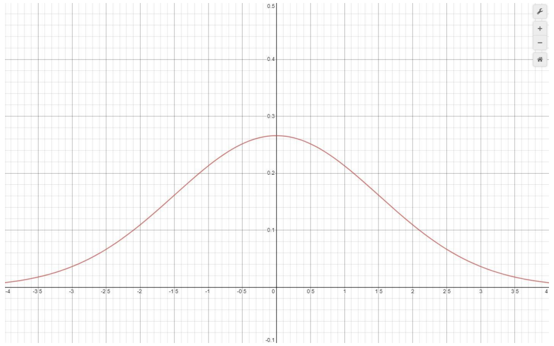
How should `random_sample` be defined?



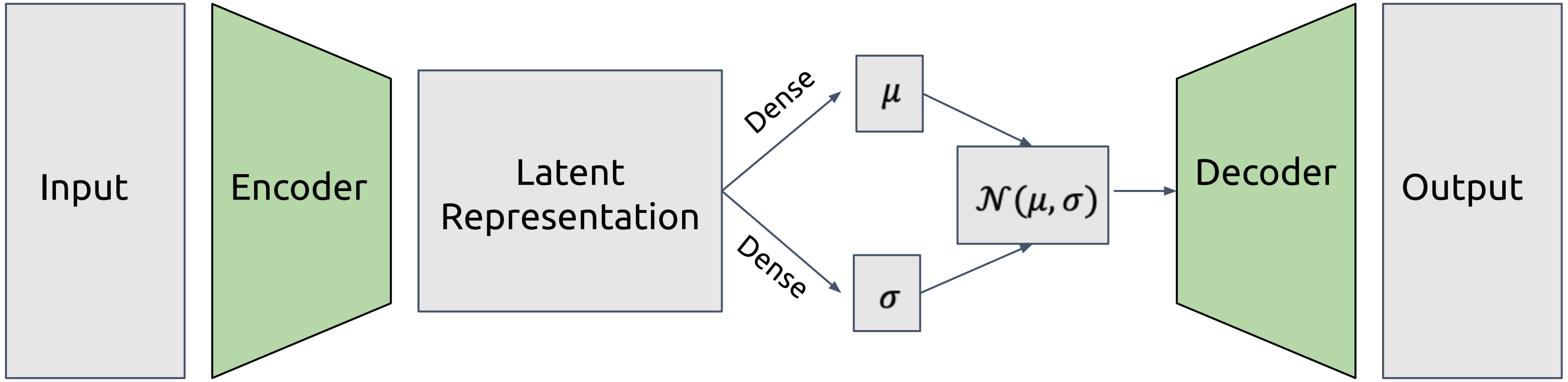
$\mu = 0$
$\sigma = 1$



$\mu = 1$
$\sigma = 1$



$\mu = 0$
$\sigma = 1.5$

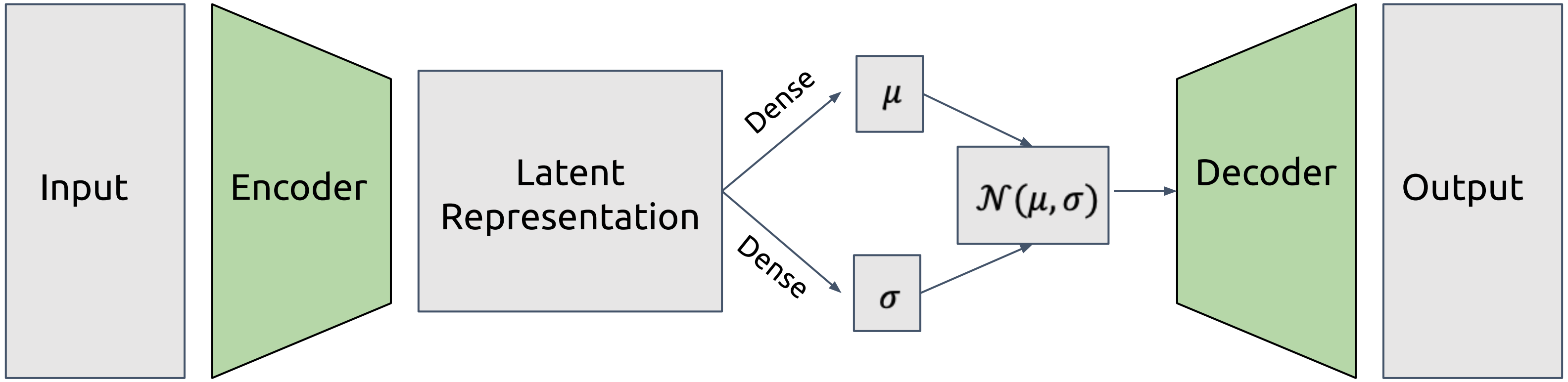


Training a VAE

Two goals:

1. Reproduce an output similar to the input (Input ≈ Output)
2. Have some variation in our output (Input $\not\approx$ Output)

- Seems like two conflicting goals!
- How do we resolve these two goals?



Weighted Combination of Losses

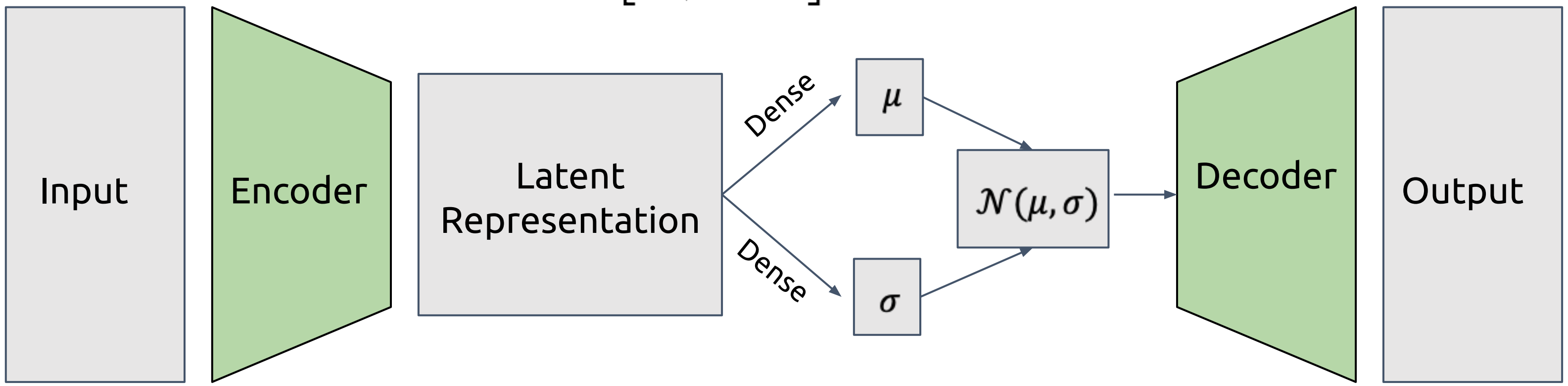
L_1 = loss associated with producing output similar to input

L_2 = loss associated with producing output with some variation to input

$$L = L_1 + \lambda L_2$$

Total Loss:

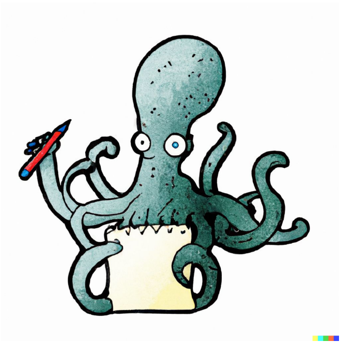
$$\lambda \in [0, \infty]$$



Recap

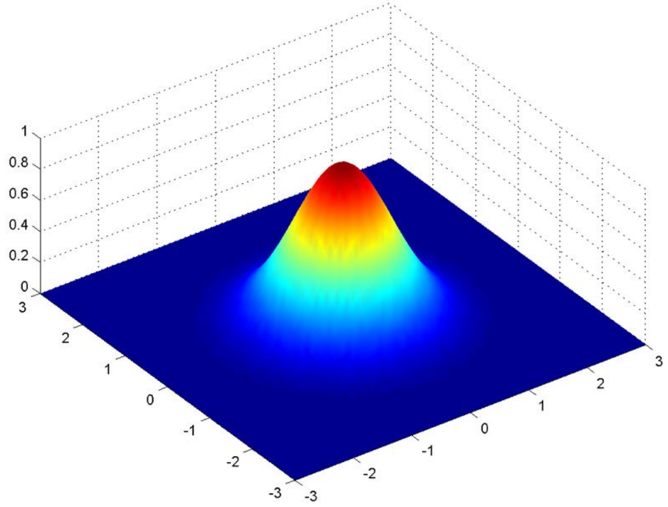
Convolutional AEs

Generative Modeling



Generative modeling – formulation and applications

Probability distributions = generative models

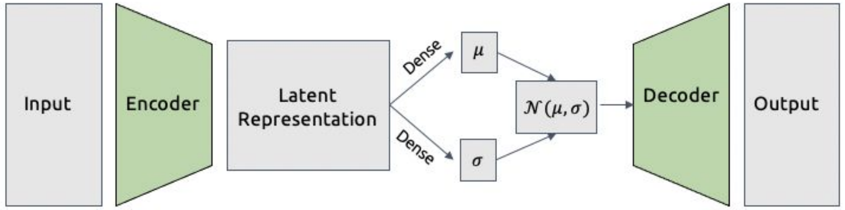


Variational Autoencoders (VAEs)

Generative modeling for complex distributions

Modifying AEs

VAE architecture

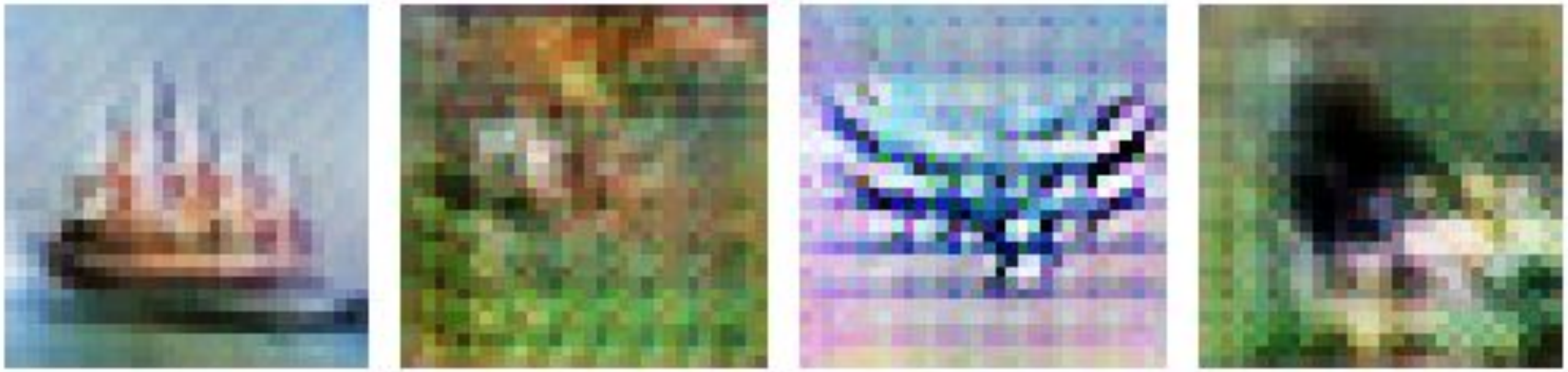


Extra material

More reading on Transpose Convolution

Caution: Checkerboard Artifacts

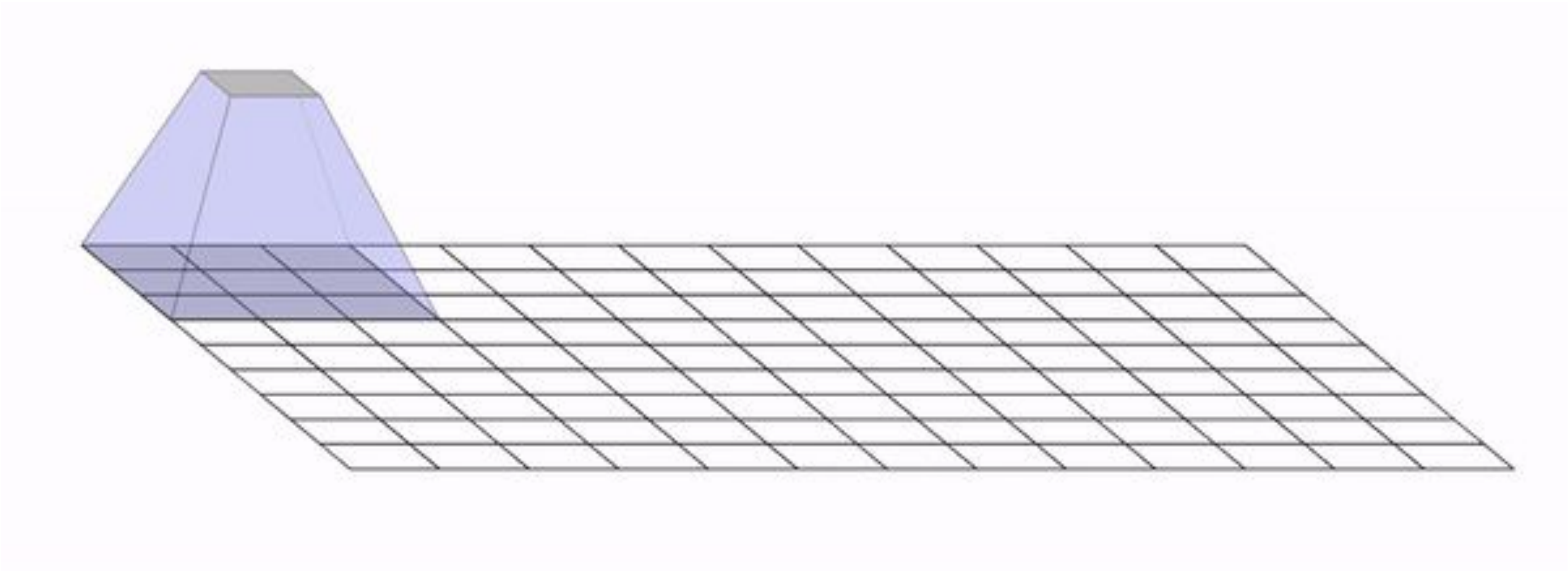
- Transpose convolution causes artifacts in output images



Great visual (and more!) from this article: https://distill.pub/2016/deconv-checkerboard/

Caution: Checkerboard Artifacts

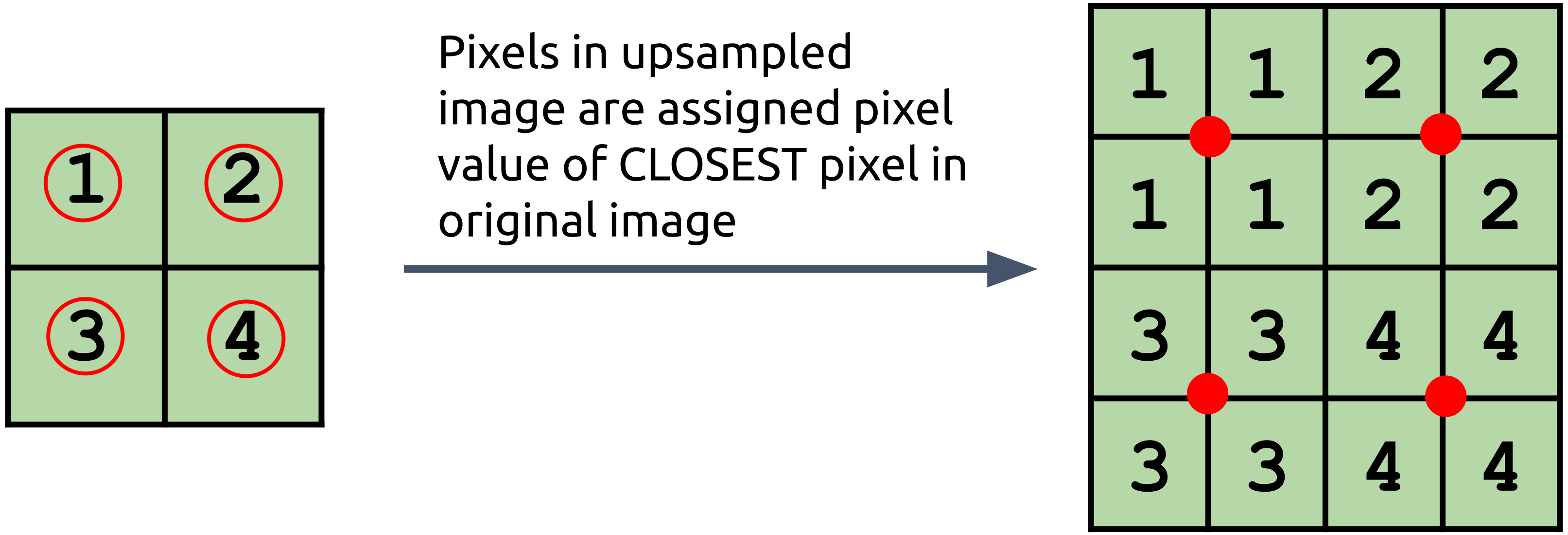
- Transpose convolution causes artifacts in output image
- Why? Some pixels get written to more often than others
- Is there a better way to upsample?



Great visual (and more!) from this article: https://distill.pub/2016/deconv-checkerboard/

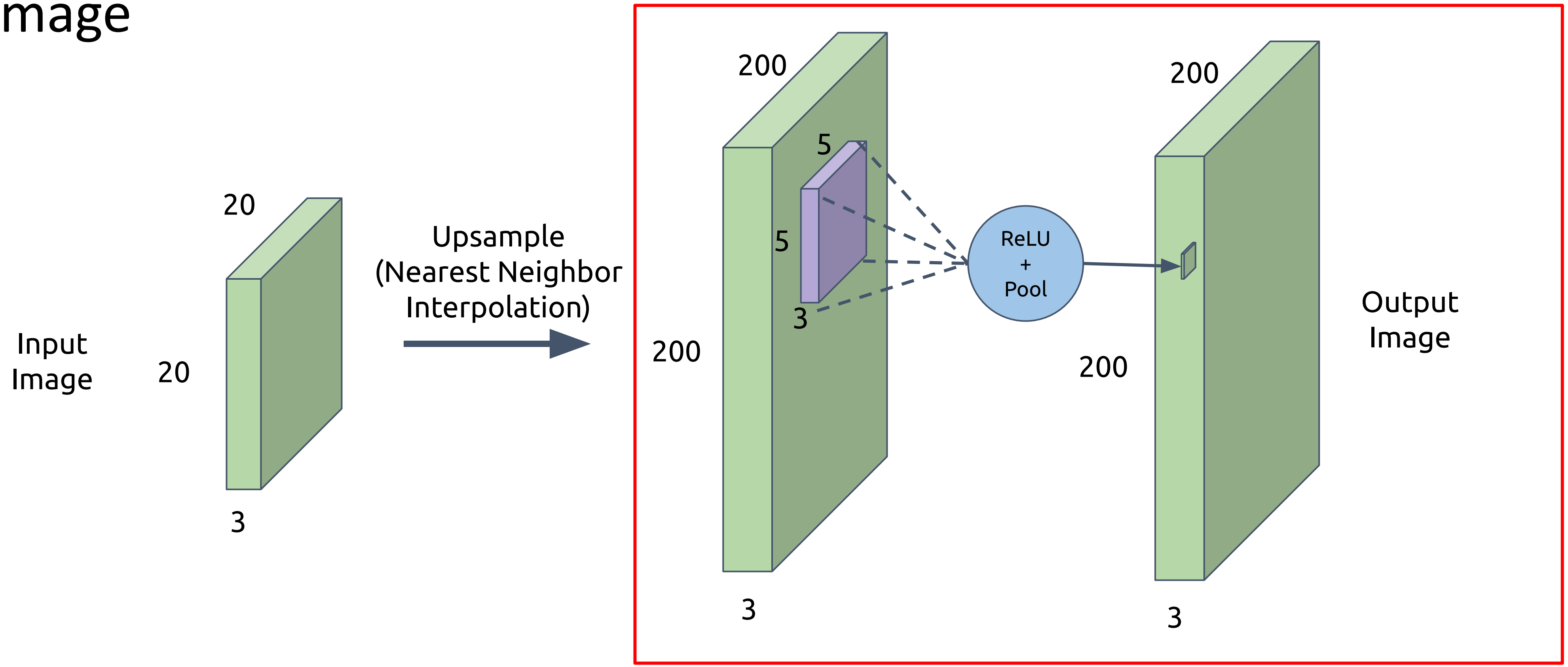
Eliminating checkerboard artifacts

Step 1: Upsample using nearest neighbor interpolation:



Eliminating checkerboard artifacts

Step 2: Perform a convolution with SAME padding on the upsampled image

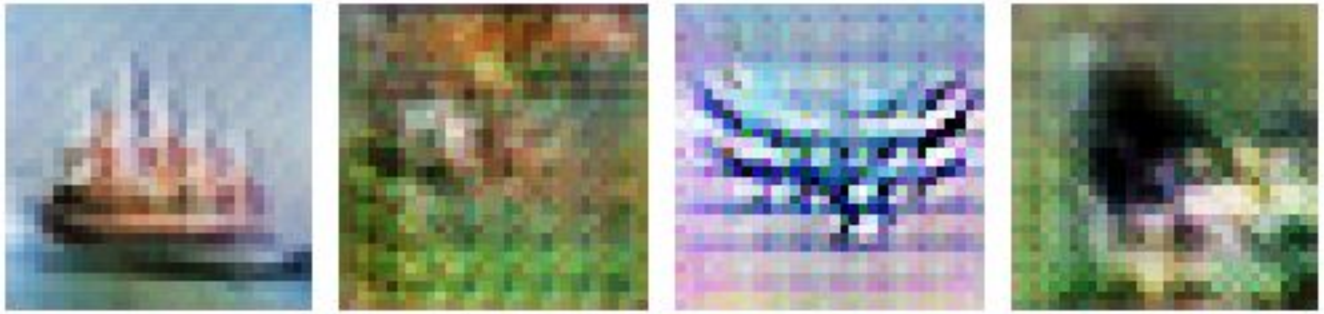


Dealing With it in Tensorflow

```
# Layer to upsample the image by a factor of 5 in x and y using nearest
# neighbor interpolation
tf.keras.layers.UpSampling2D(size=(5, 5), interpolation='nearest')


# Do a convolutional layer on the result
tf.keras.layers.Conv2D(filters = 1, kernel_size = (10,10), padding = "SAME")
```

Checkerboard Artifacts Resolved



With Transpose Convolution

With Resize + Convolution

Great visual (and more!) from this article: https://distill.pub/2016/deconv-checkerboard/