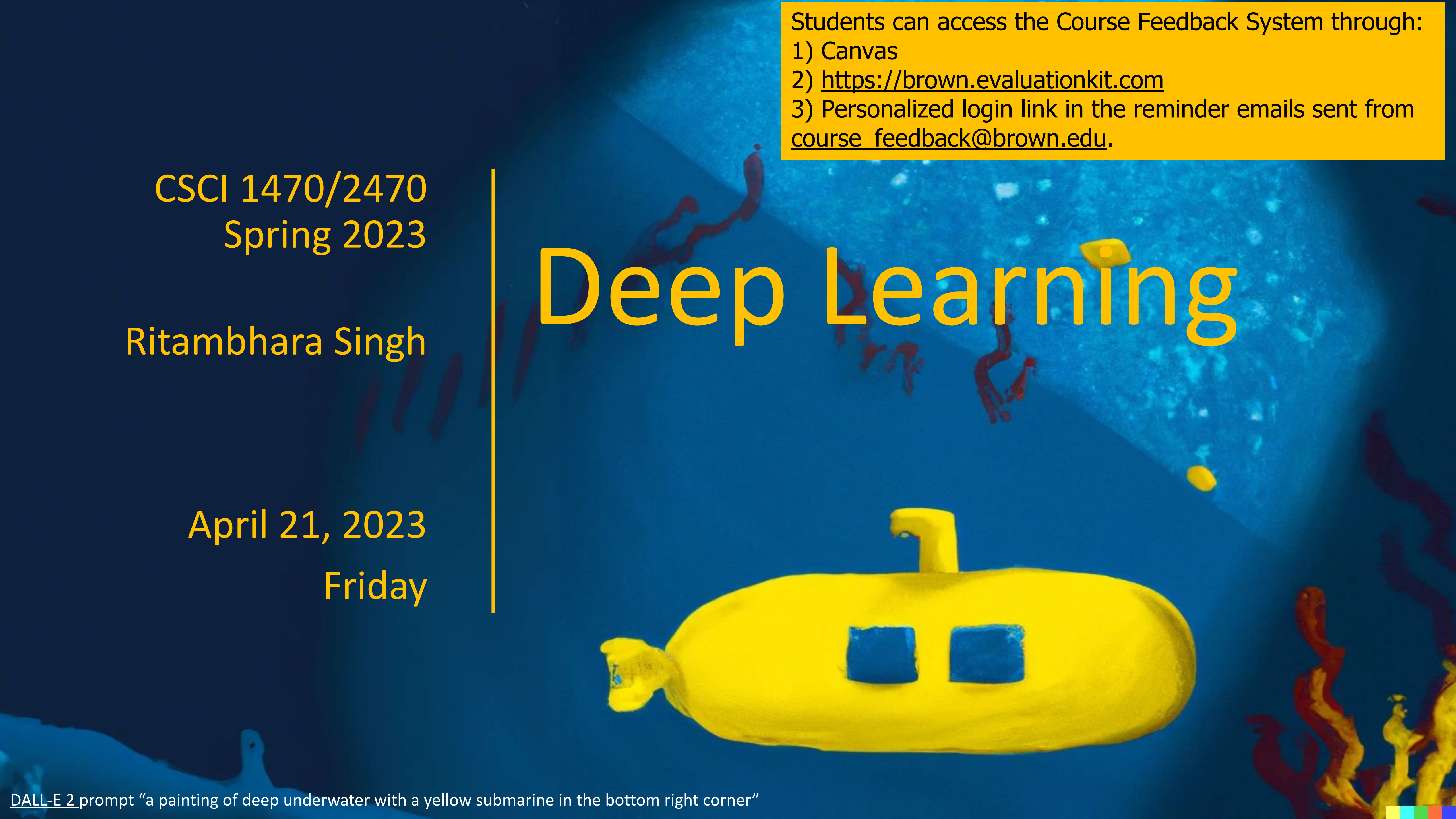
Students can access the Course Feedback System through:
1) Canvas
2) https://brown.evaluationkit.com
3) Personalized login link in the reminder emails sent from course_feedback@brown.edu.

CSCI 1470/2470
Spring 2023

Ritambhara Singh

April 21, 2023
Friday

Deep Learning



DALL-E 2 prompt “a painting of deep underwater with a yellow submarine in the bottom right corner”

Review: Q-value and V-value Tables (made up)

	Action #1	Action #2
State #1	0	-1
State #2	0.1	1
State #3	-1	-10
State #4	0	1.9
State #5	10	0
State #6	-10	-10

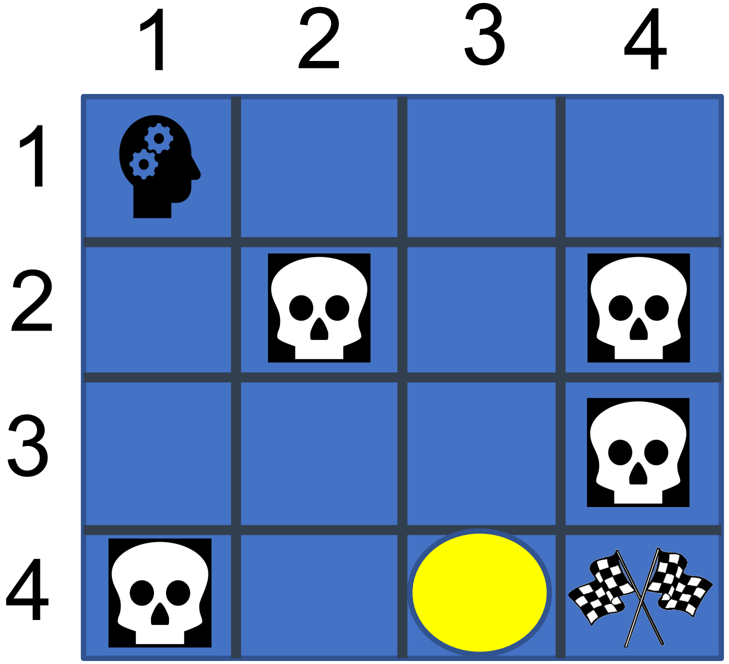
State	Value
State #1	0
State #2	1
State #3	-1
State #4	1.9
State #5	10
State #6	-10

Review: Value iteration pseudocode

1. For all s, set $V(s) := 0$.
2. Repeat until convergence:
   1. For all s:
      1. For all a, set $Q(s,a) \coloneqq \sum_{s' \in S} T(s,a,s')[R(s,a,s') + \gamma V(s')]$
      2. $V(s) := max_a\, Q(s,a)$
3. Return Q

Review: Frozen Lake – final value table & optimal policy

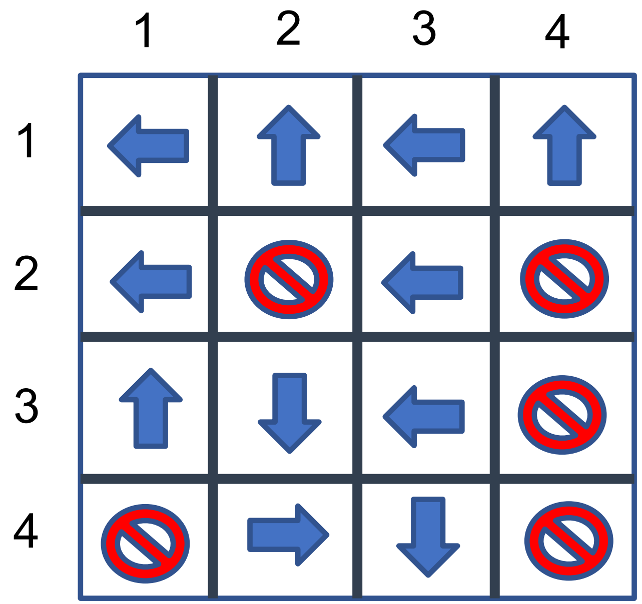
Environment



Example Final Value Table

	1	2	3	4
1	0.068	0.061	0.074	0.055
2	0.092	0	0.112	0
3	0.145	0.247	0.3	0
4	0	0.38	0.639	0

Final Policy



Organizing RL problems/algorithms

	Know T and R	Don't know T and R
Simple/discrete	Value iteration	Q-Learning
Complex/continuous		Deep Q-Networks REINFORCE Actor-Critic

For a more complete taxonomy of RL algorithms, see https://spinningup.openai.com/en/latest/spinningup/rl_intro2.html#citations-below

Tabular Q-learning

Motivation: Why Not Value Iteration?

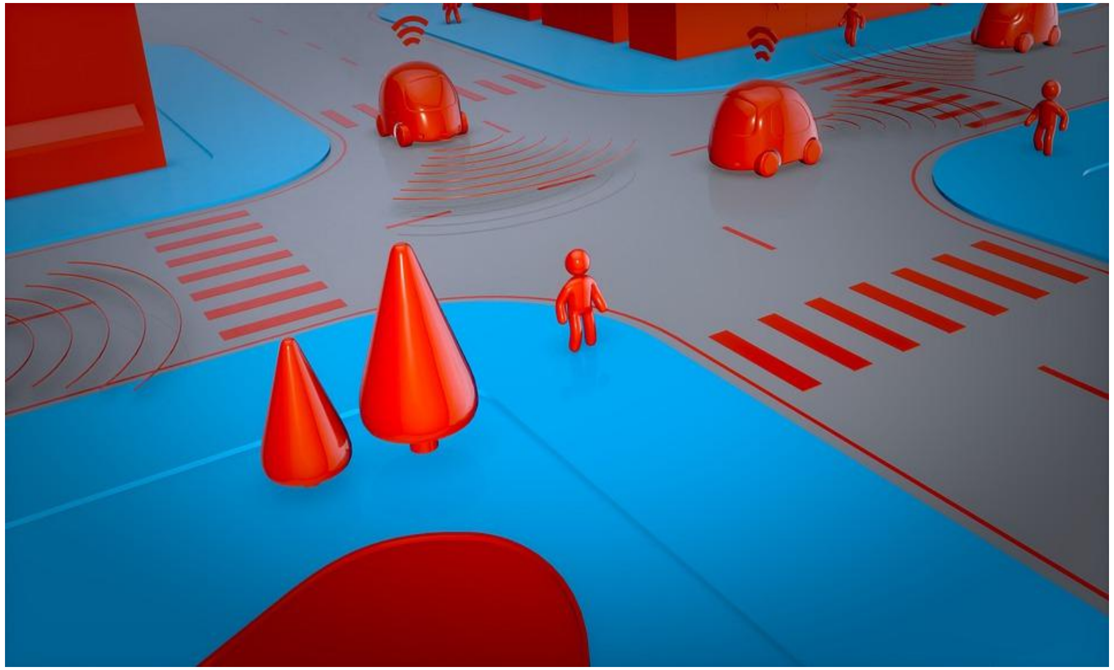
In value iteration, we assume that we know the transition and reward functions, but what if this isn't the case?

$$\{S, A, \cancel{R}, \cancel{T}, \gamma\}$$

How can we learn in this scenario?

Examples of Unknown T and/or R:

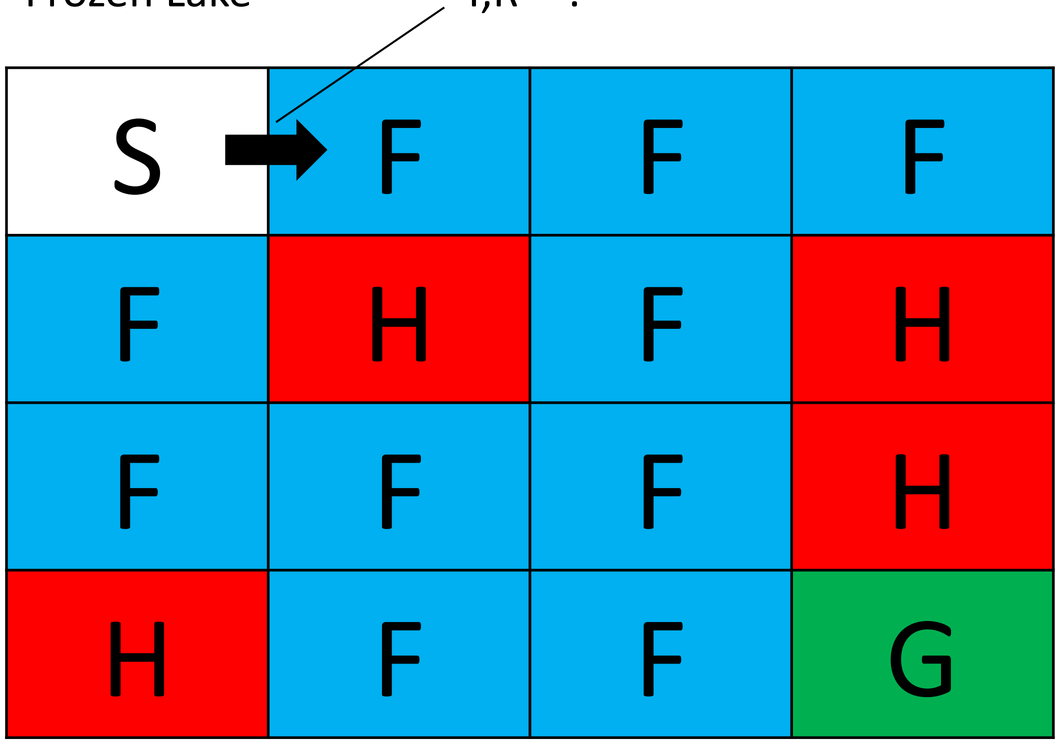
Self-Driving Cars



https://pixabay.com/illustrations/self-driving-car-autonomous-4309836/

Can’t design T for a self-driving car.
Too complex to model directly

Frozen Lake



What if we don’t know how slippery the lake is?
(i.e. T is unknown)

First Attempt

Start in Frozen Lake with no knowledge of T or R

S	F	F	F
F	H	F	H
F	F	F	H
H	F	F	G

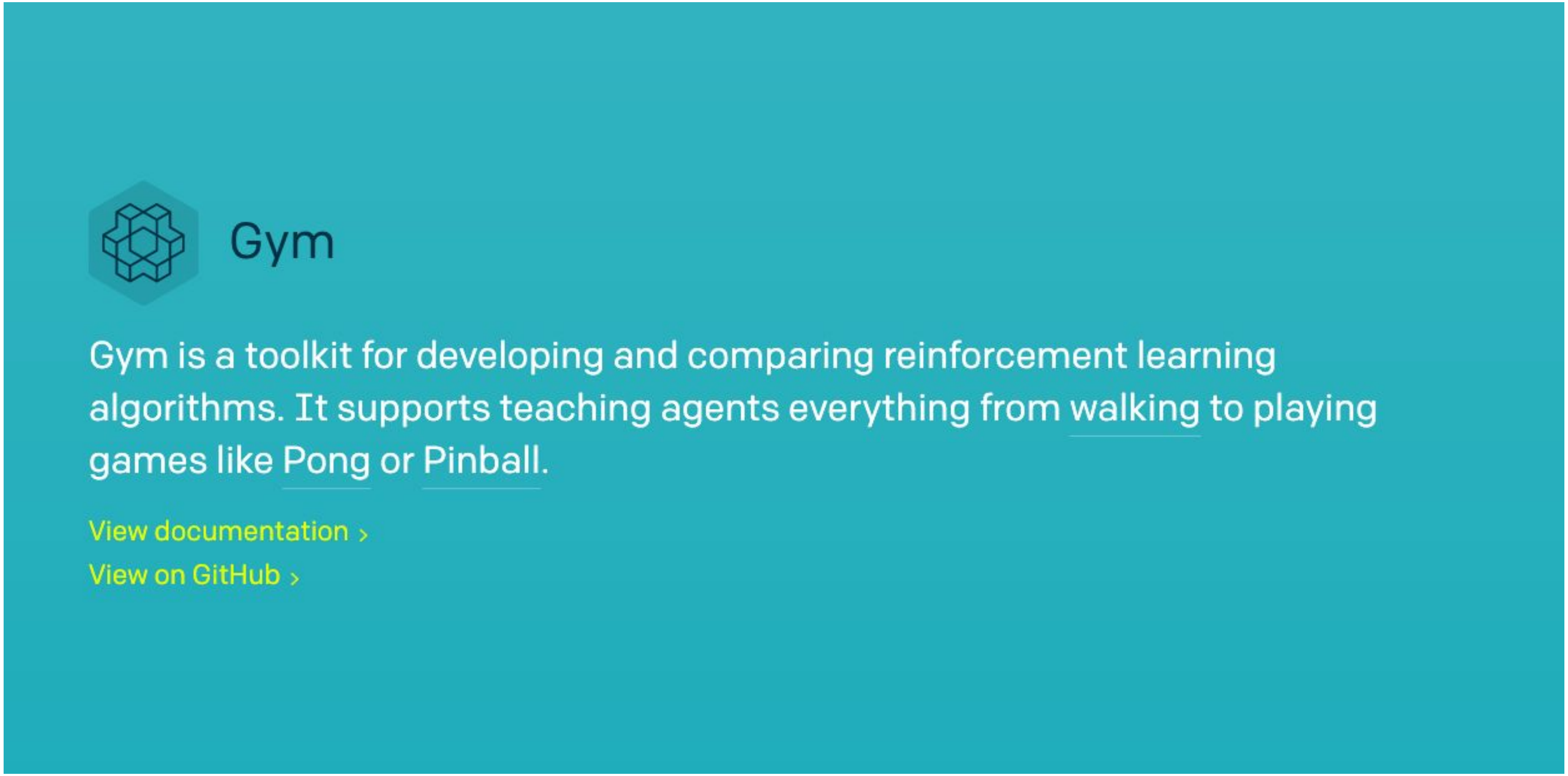
Take random actions to estimate T and R

Run value iteration using estimates of T and R

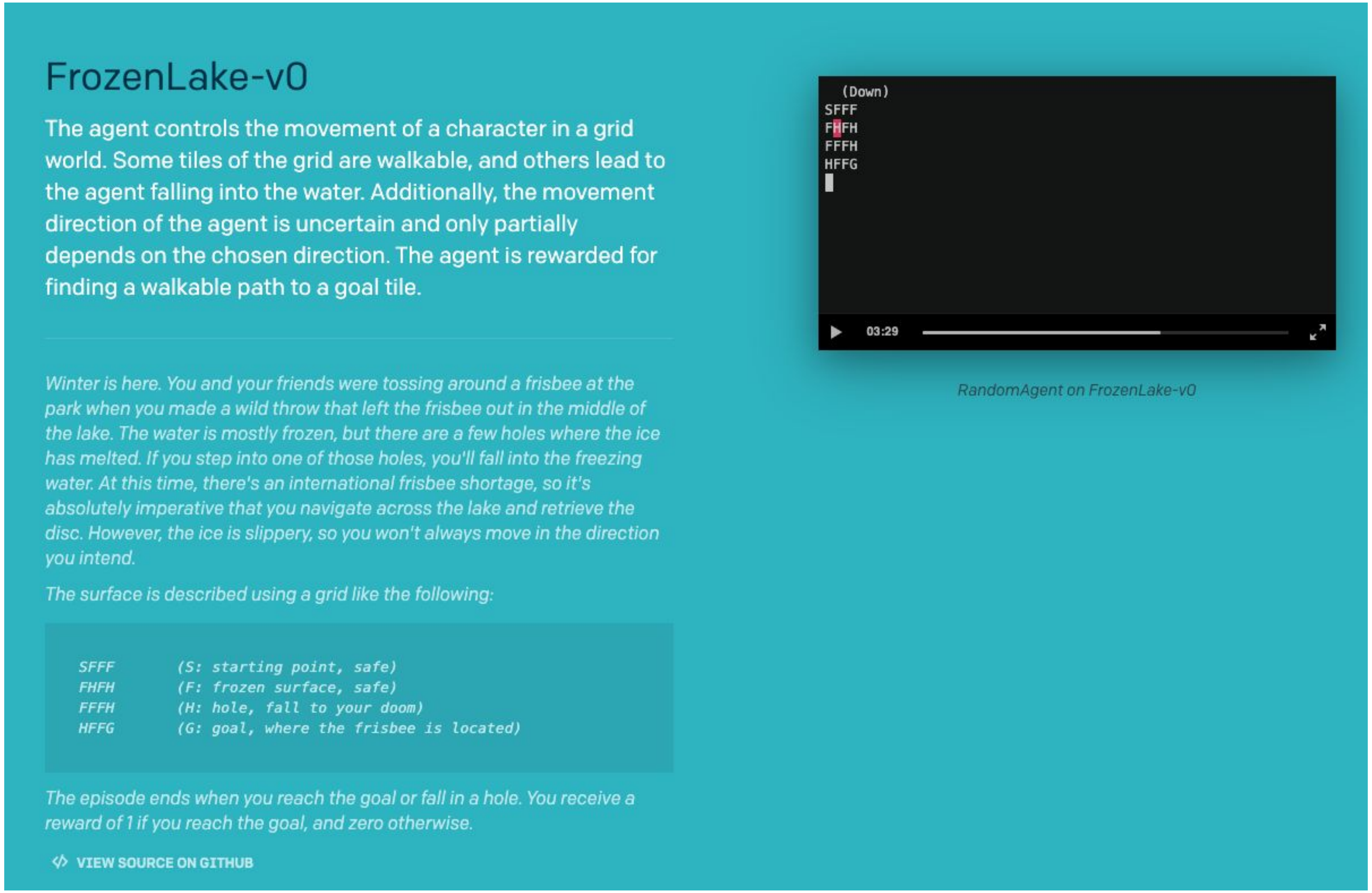
$$Q(s,a) = \sum_{s'} T(s,a,s')R(s,a,s') + \gamma V(s')$$

$$V(s) = \text{argmax}_a(Q(s,a))$$

OpenAI Gym (https://gym.openai.com/)



Frozen Lake in OpenAI Gym



FrozenLake-v0

The agent controls the movement of a character in a grid world. Some tiles of the grid are walkable, and others lead to the agent falling into the water. Additionally, the movement direction of the agent is uncertain and only partially depends on the chosen direction. The agent is rewarded for finding a walkable path to a goal tile.

Winter is here. You and your friends were tossing around a frisbee at the park when you made a wild throw that left the frisbee out in the middle of the lake. The water is mostly frozen, but there are a few holes where the ice has melted. If you step into one of those holes, you'll fall into the freezing water. At this time, there's an international frisbee shortage, so it's absolutely imperative that you navigate across the lake and retrieve the disc. However, the ice is slippery, so you won't always move in the direction you intend.

The surface is described using a grid like the following:

```
SFFF       (S: starting point, safe)
FHFH       (F: frozen surface, safe)
FFFH       (H: hole, fall to your doom)
HFFG       (G: goal, where the frisbee is located)
```

The episode ends when you reach the goal or fall in a hole. You receive a reward of 1 if you reach the goal, and zero otherwise.

VIEW SOURCE ON GITHUB

RandomAgent on FrozenLake-v0

Frozen Lake ‘Wandering’ Demo

https://colab.research.google.com/drive/1yBCDzAXlu9j0A2aT8fH1zm7beBgtB9yY#scrollTo=sI_x2TsCp15L

Problems with This Approach

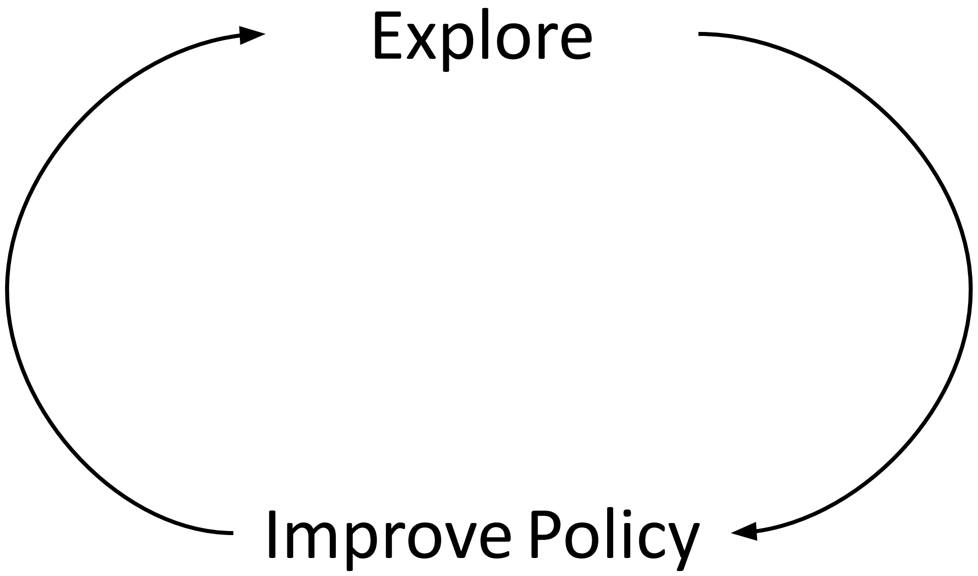
S 32709	F 13151	F 5951	F 2868
F 12335	H 7095	F 1760	H 1141
F 4646	F 1579	F 1015	H 252
H 1380	F 643	F 501	G 132

Values are the number of times each state was visited over 10000 episodes run in OpenAI Gym

So, what is the issue here?

We're extremely unlikely to reach the goal state through random wandering, so our estimates of T and R will probably be bad

How to Improve?



We can interleave policy improvement with wandering
(Get better at exploring ***by exploring***)

But how do we improve our policy?

Q-Learning

- Every time we take an action, use the observed reward to update our estimate of Q
- $V(s)$ is still $\max_a Q(s, a)$, so it only matters how we update Q

S	F	F	F
F	H	F	H
F	F	F	H
H	F	F	G

$$Q^{\pi}(s,a) = E[R(s,a,s') + \gamma V^{\pi}(s')]$$

$$Q^{\pi}(s,a) = \sum_{s' \in S} P(s'|s,a)[R(s,a,s') + \gamma V^{\pi}(s')]$$

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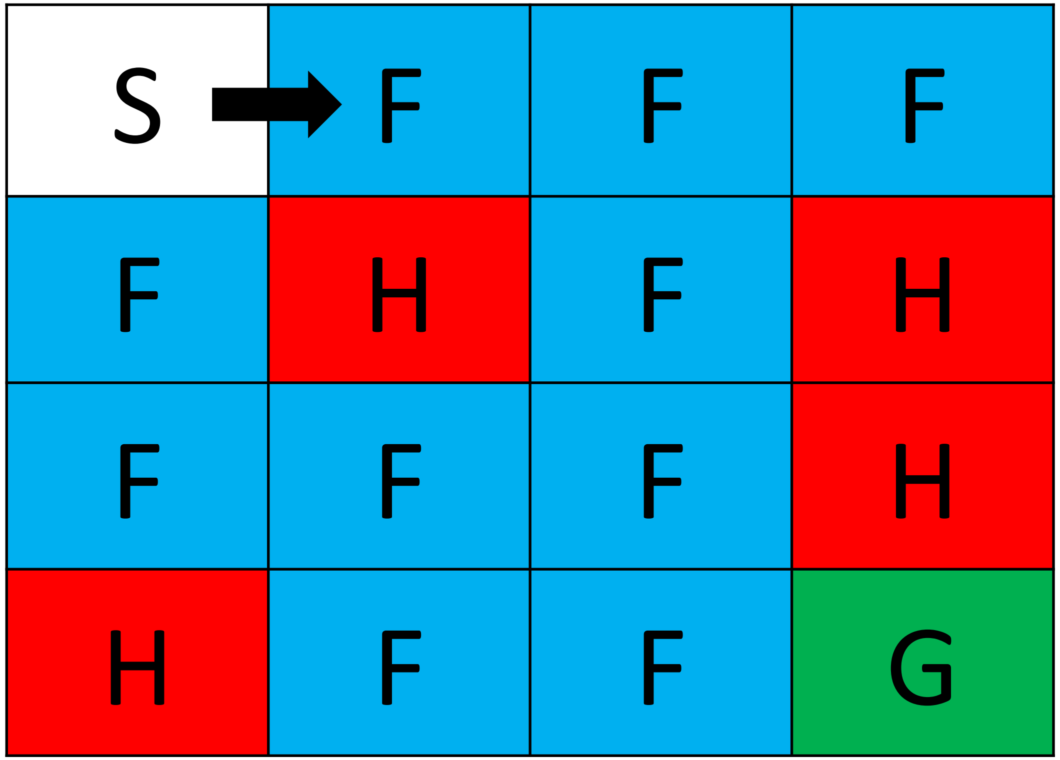

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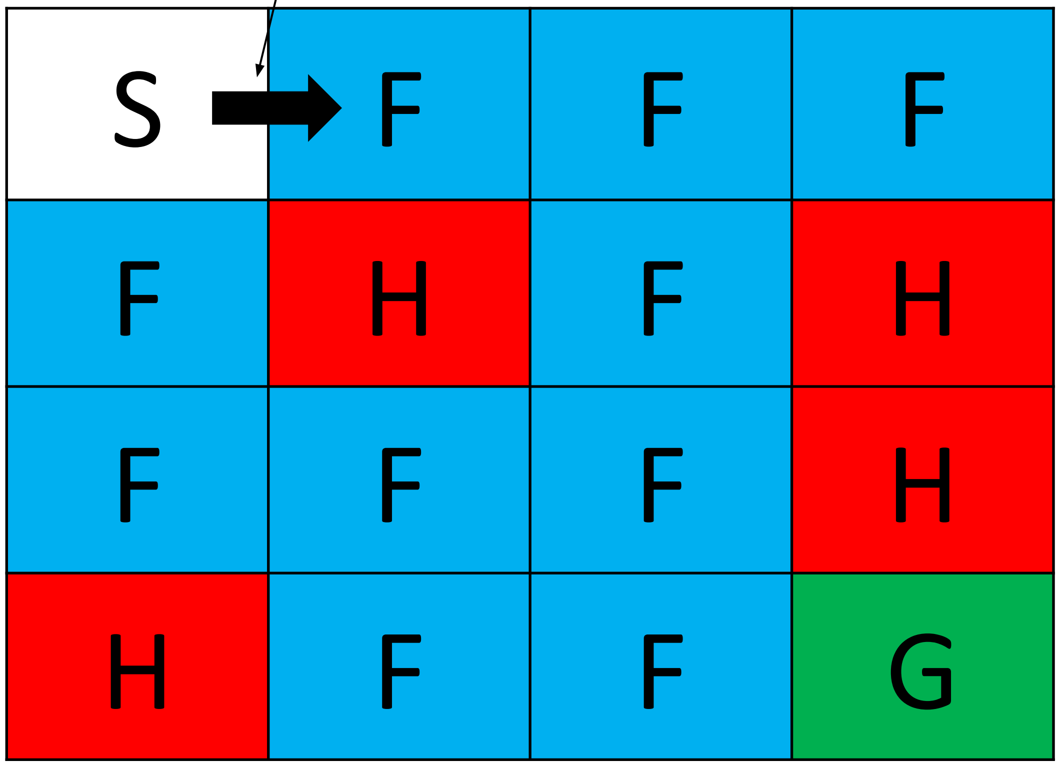
Q-Learning

- Every time we take an action, use the observed reward to update our estimate of Q
- $V(s)$ is still $\max_a Q(s, a)$, so it only matters how we update Q

Use Q to pick action a_0. Observe transition and get reward r_0. Use r_0 to update $Q(s_0, a_0)$ immediately



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Q-Learning

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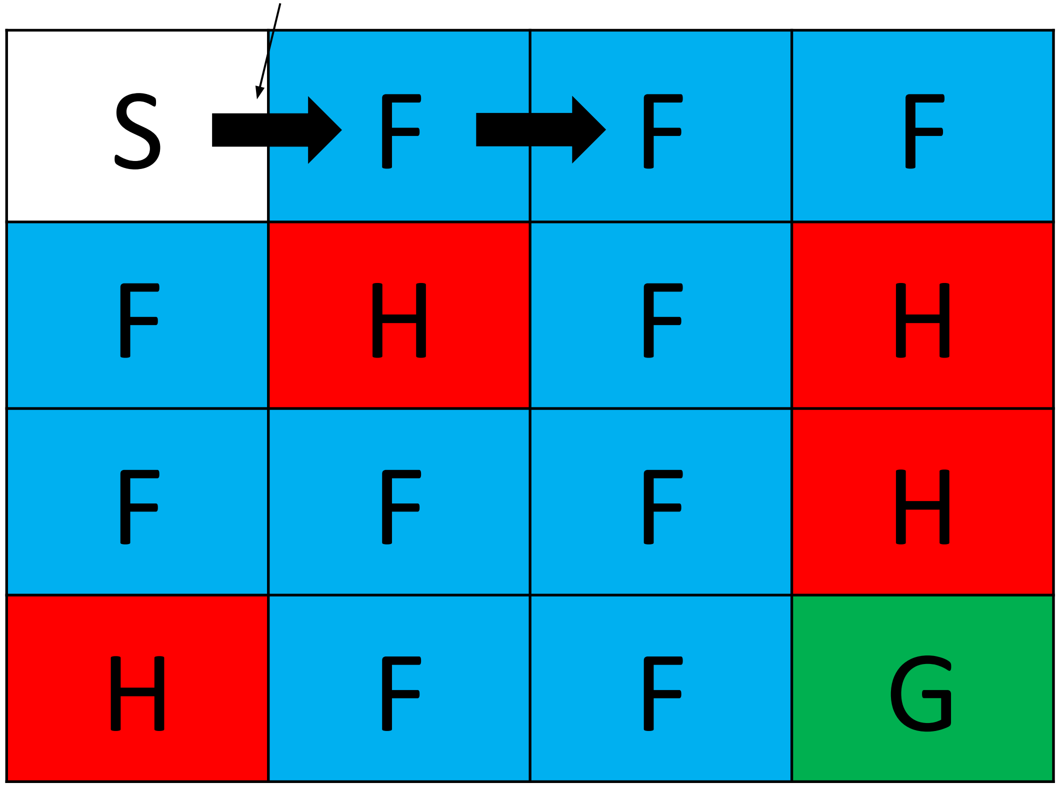
Use Q to pick action a_0. Observe transition and get reward r_0. Use r_0 to update $Q(s_0, a_0)$ immediately



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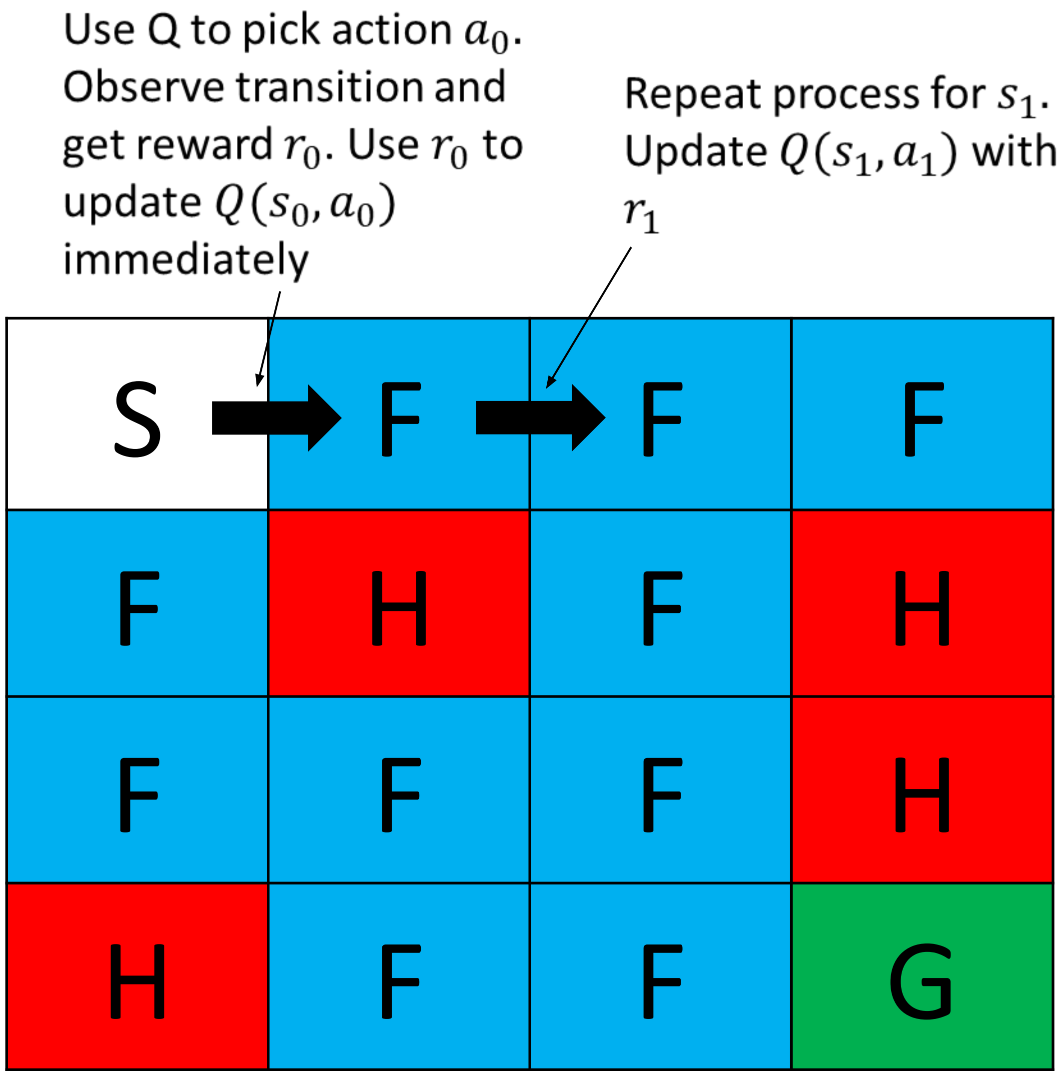

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$$Q^\pi(s,a) = \sum_{s' \in S} P(s'|s,a)[R(s,a,s') + \gamma V^\pi(s')]$$

How do we Update Q?

Basic Strategy:

Take a weighted average of our old Q estimate with our new Q estimate

$$Q(s,a) = (1-\alpha)Q_{old}(s,a) + \alpha Q_{new}(s,a)$$

Where the hyperparameter α controls how quickly we learn

But what should $Q_{new}(s,a)$ be?

Determining $Q_{new}(s, a)$

In value iteration, we took an expectation over all possible next states and actions to get a new estimate for $Q(s, a)$

$$Q(s,a) = \sum_{s'} T(s,a,s')(R(s,a,s') + \gamma V(s'))$$

How do we change this?

In Q-Learning, we only take one action and see one new state, and use that one transition to update our estimate for $Q(s, a)$, so our update rule becomes

$$Q(s,a) = R(s,a,s') + \gamma V(s')$$

i.e. the reward we observed for moving into state s', plus whatever our current value function estimate thinks is the value of being in state s'

The Q-Learning Update Rule

Any questions?

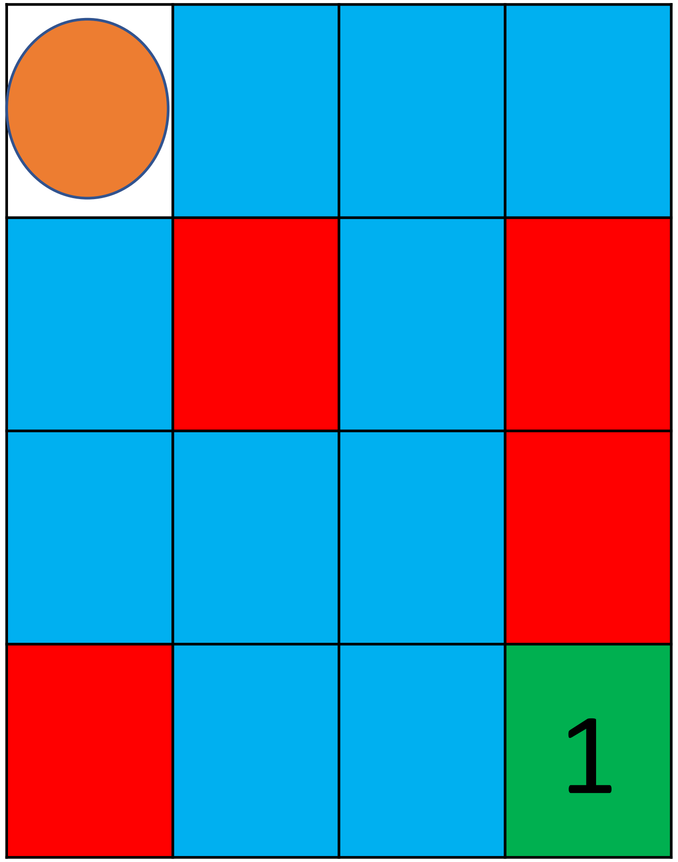
Combining our new estimate for $Q(s,a)$ with the weighted average equation, the final update rule for Q-Learning becomes

$$Q(s,a) = (1-\alpha)Q_{old}(s,a) + \alpha Q_{new}(s,a)$$

$$\Downarrow$$

$$Q(s,a) = (1-\alpha)Q(s,a) + \alpha(R(s,a,s') + \gamma V(s'))$$

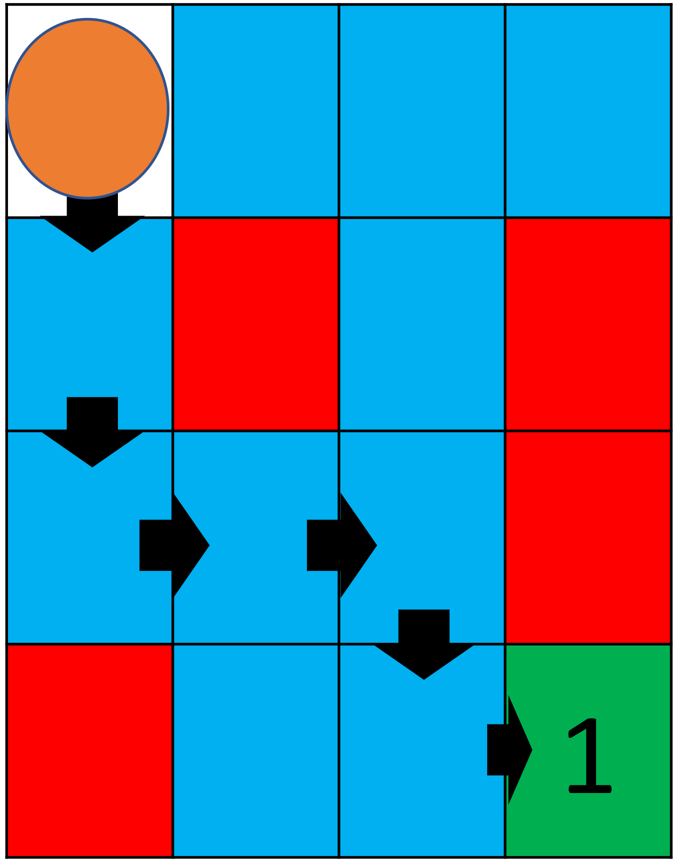
Q-Learning in Frozen Lake



- $\alpha = .5, \gamma = .99$
- Values in the grid are Q(s,a)
- Blank quadrants have a Q-value of 0

- Initially, the agent is acting randomly, because all states have a value of zero
  - It would likely fall into the holes many times before reaching the goal state for the first time

Q-Learning in Frozen Lake



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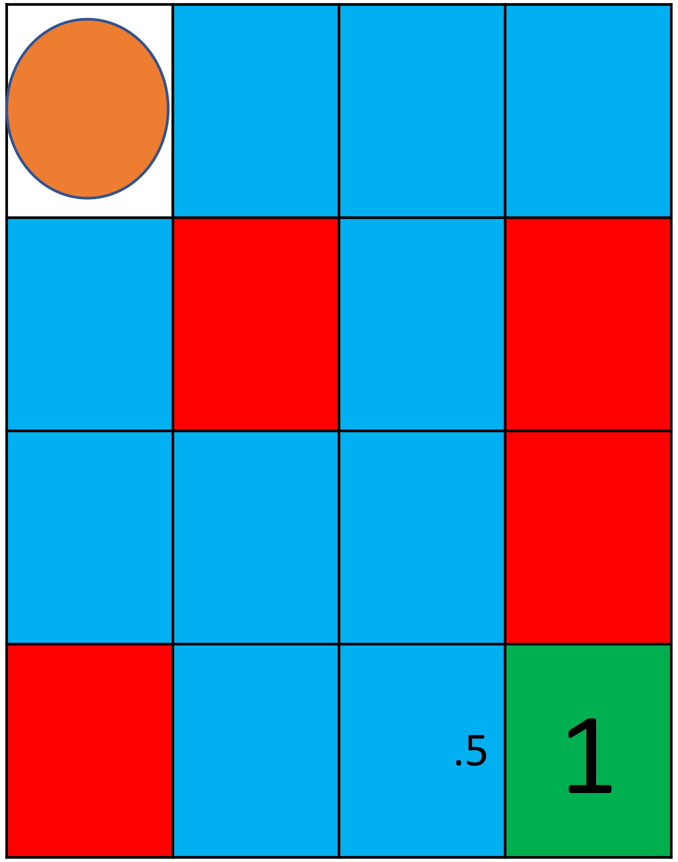
- At every transition in this path(each <s,a,r,s’> tuple), the agent sees a reward of zero up until the final one, so none of the Q-estimates change except the state before the goal
  - $Q(s, a) = (1 - \alpha)0 + \alpha(0) = 0$

Q-Learning in Frozen Lake



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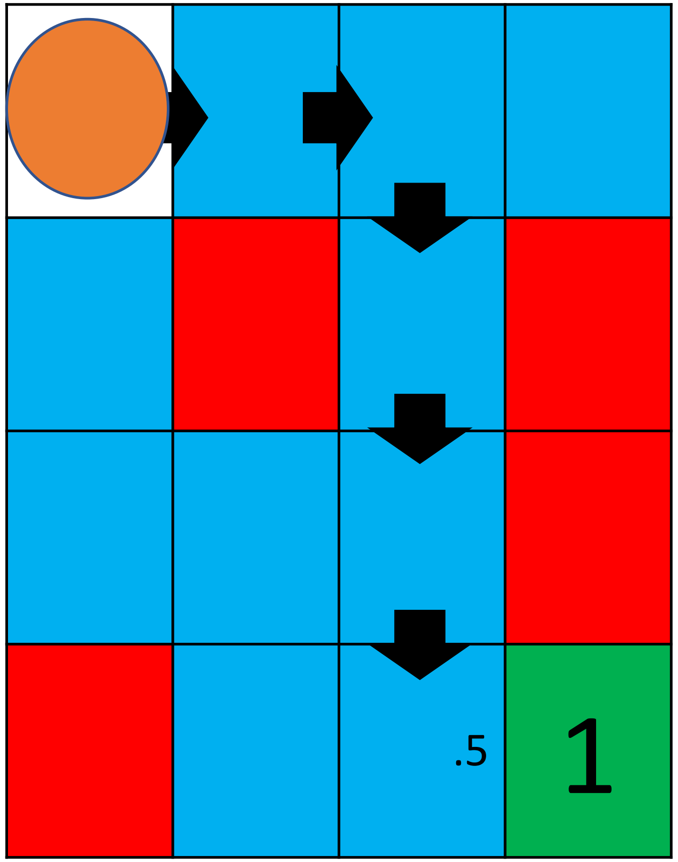
- At every transition in this path(each <s,a,r,s'> tuple), the agent sees a reward of zero up until the final one, so none of the Q-estimates change except the state before the goal
  - $Q(s,a) = (1-\alpha)0 + \alpha(0) = 0$
- The last transition has a positive reward though, so the previous state action pair is changed
  - $Q(s,a) = (1-\alpha)0 + \alpha(1) = .5$

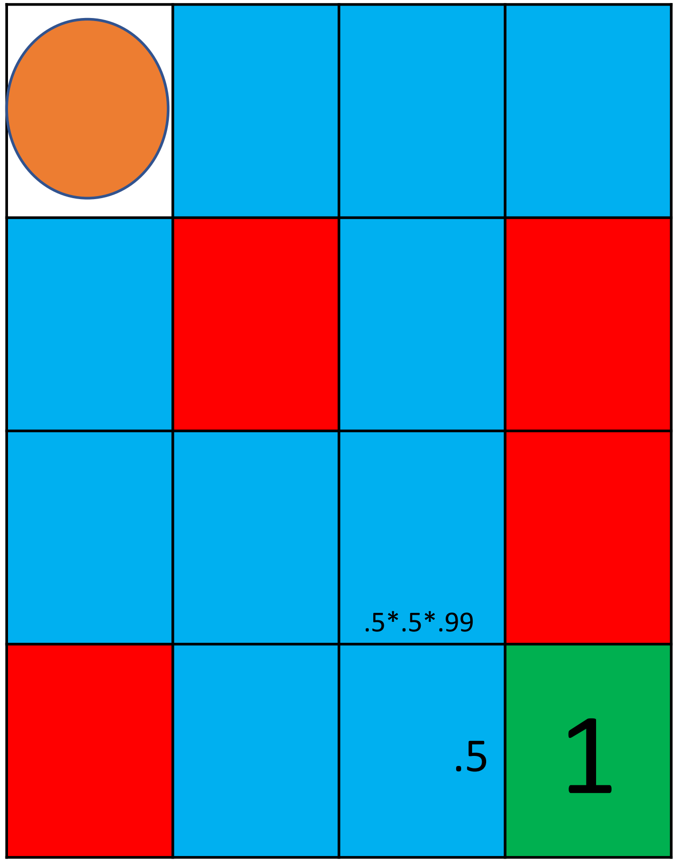
Q-Learning in Frozen Lake



- $\alpha = .5, \gamma = .99$
- Values in the grid are Q(s,a)
- Blank quadrants have a Q-value of 0

- In another episode, the agent will still act randomly(since most values are still zero), unless it reaches the state next to the goal, which now has a positive value.
  - Assume it got lucky once again

Q-Learning in Frozen Lake



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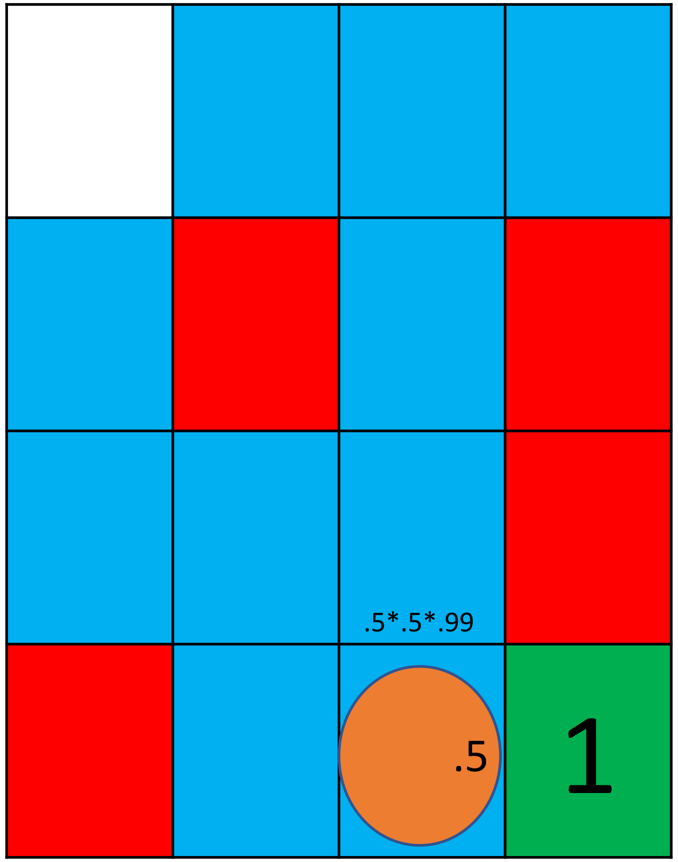
  - Assume it got lucky once again
- When that happens, the previous state-action pair will be updated using the value of the state next to the goal
  - $Q(s,a) = (1 - \alpha)0 + \alpha(0 + \gamma * .5)$ = .5*.5*.99

Q-Learning in Frozen Lake



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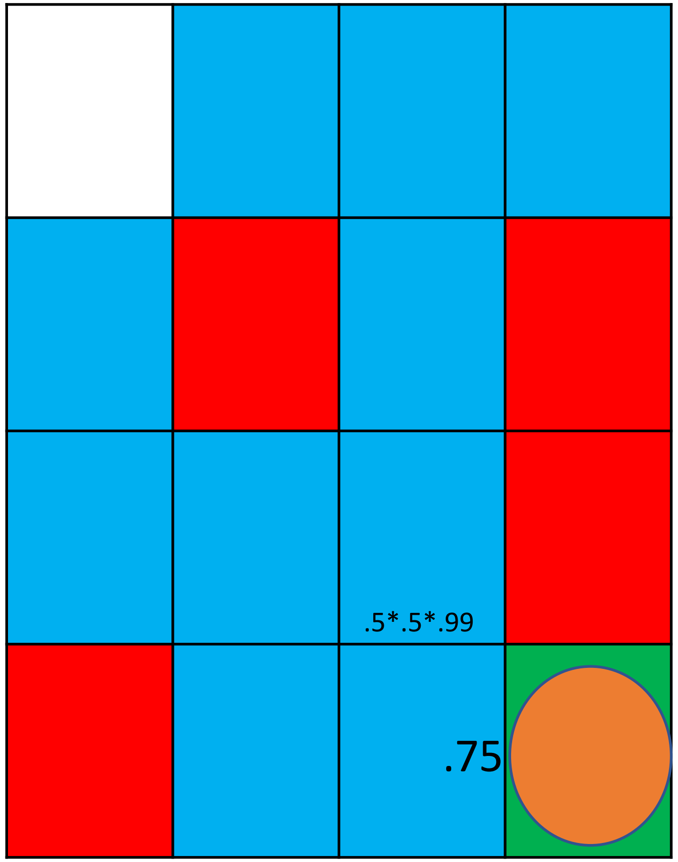
  - $Q(s, a) = (1 - \alpha)0 + \alpha(0 + \gamma * .5)$ = .5*.5*.99
- When choosing its next action from the state next to the goal, the agent will choose $\text{argmax}_a Q(s, a)$, picking the action corresponding to the value of .5 (since all the other action-values for that state are zero).

Q-Learning in Frozen Lake



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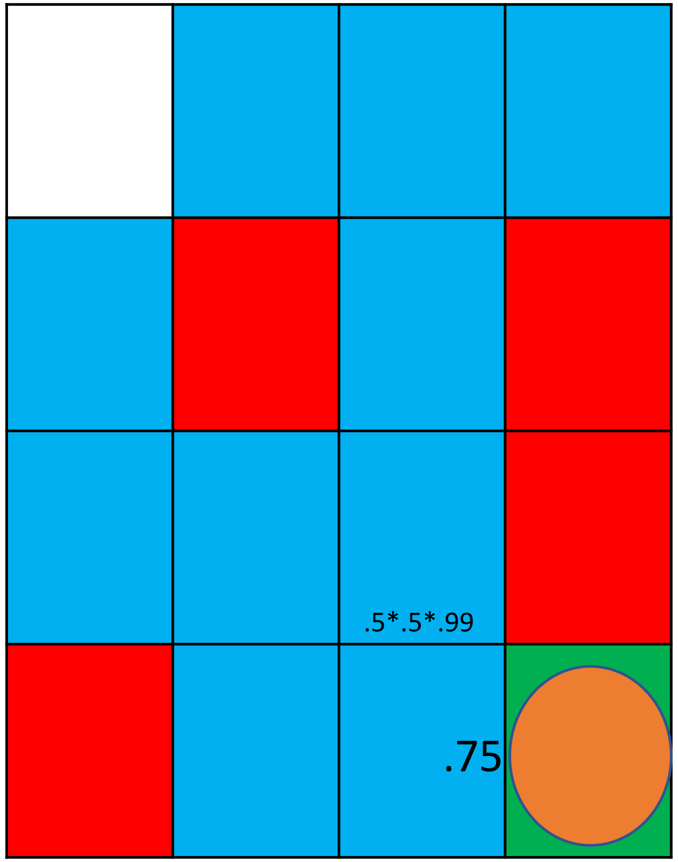
- The value of the previous state is then updated again
  - $Q(s,a) = (1 - \alpha).5 + \alpha(1)$ = .75

Q-Learning in Frozen Lake



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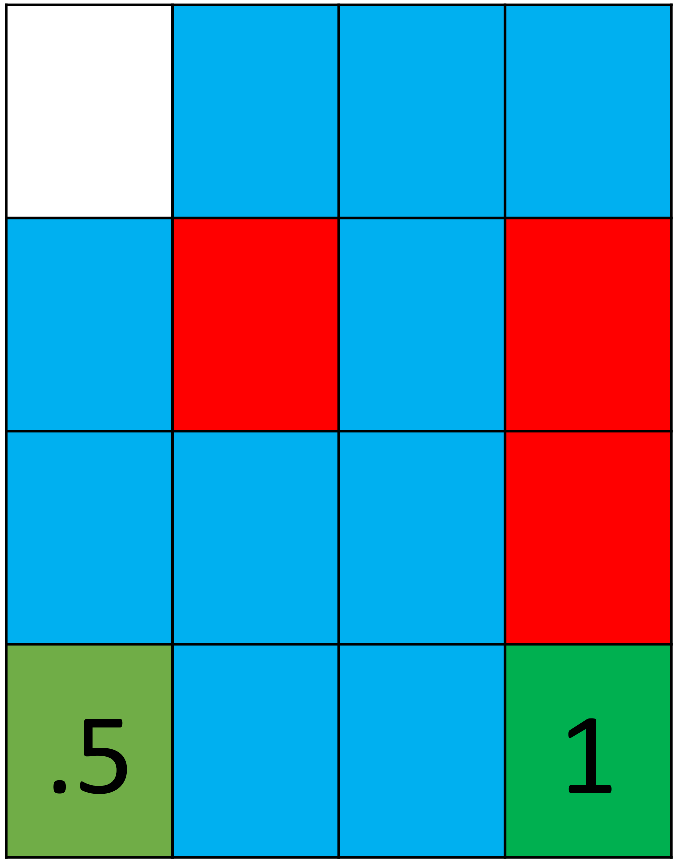
- The value of the previous state is then updated again
  - $Q(s,a) = (1-\alpha).5 + \alpha(1)$ = .75
- This process repeats over and over again until the Q-values converge to $Q^*(s,a)$, the optimal Q-values.

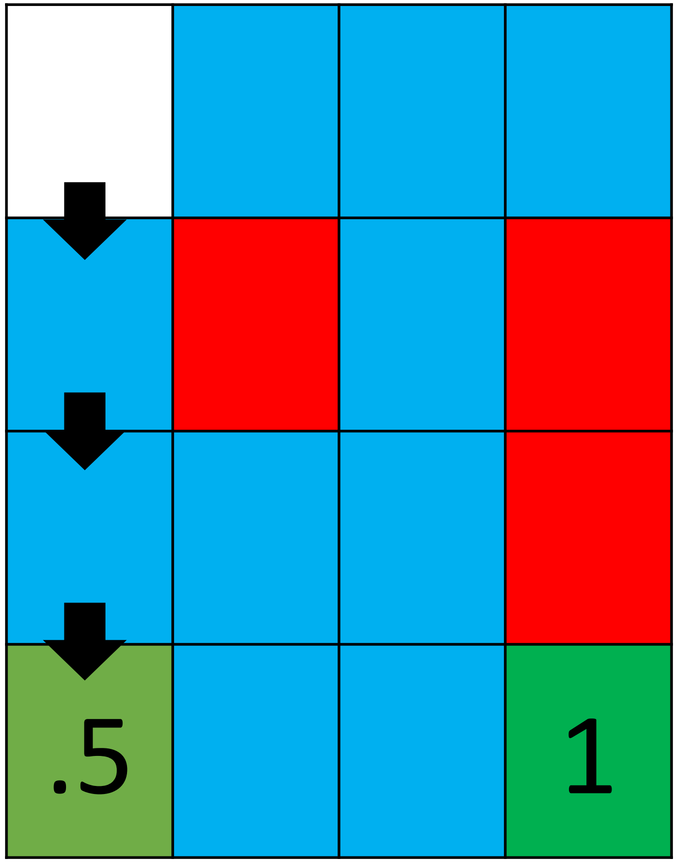
Problem: Exploration/Exploitation

- But what if Frozen Lake had two goal states?



Problem: Exploration/Exploitation

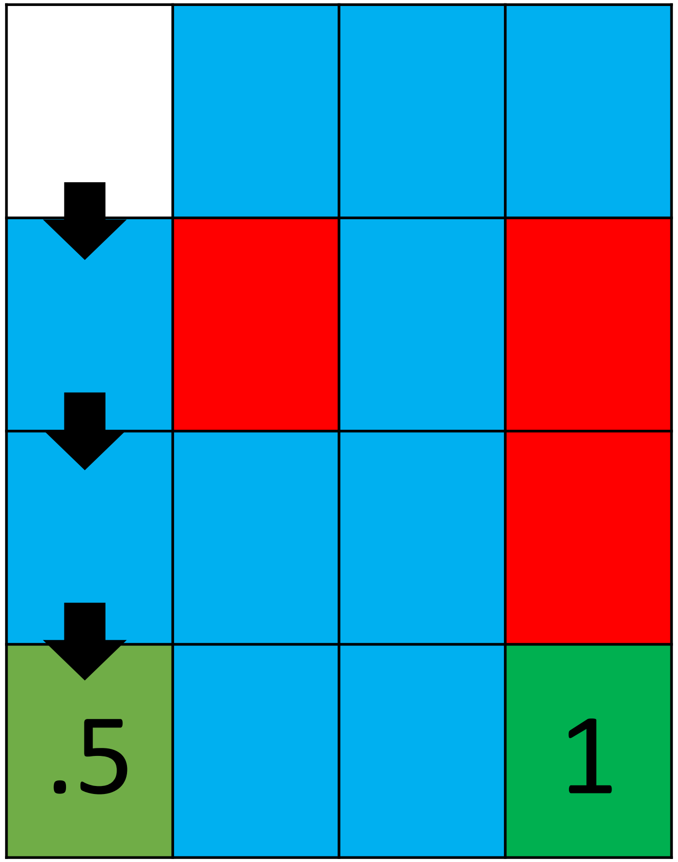
- But what if Frozen Lake had two goal states?
  - Q-Learning could learn a path to the .5 reward without ever getting to the 1 reward state.
  - Converges to a suboptimal solution



Problem: Exploration/Exploitation

Any ideas?

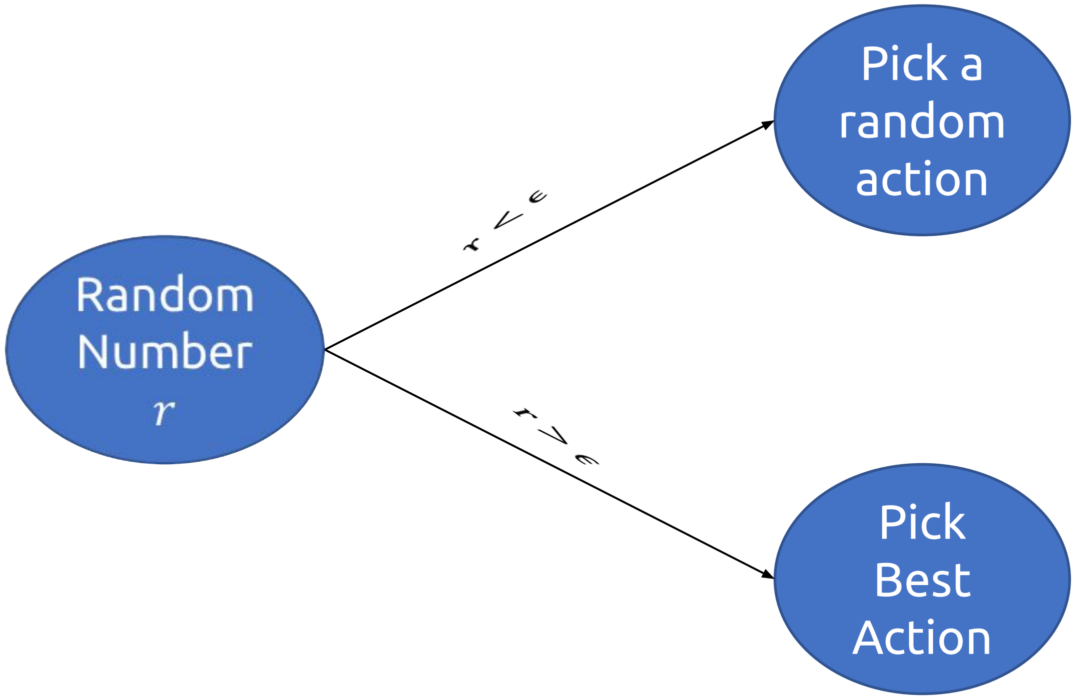
- But what if Frozen Lake had two goal states?
  - Q-Learning could learn a path to the .5 reward without ever getting to the 1 reward state.
  - Converges to a suboptimal solution
- How can we balance exploiting knowledge we already have with exploring unseen parts of the state space?



Solution: Epsilon-Greedy Policies

- Instead of always following our estimate of Q, we instead take random actions ϵ percent of the time, where ϵ is a hyperparameter
- We can also decrease ϵ over time, as our estimates of Q improve
  - Ex: after each episode, set $\epsilon = \frac{E}{(E+i)}$, where i is the number of

Action Selection Procedure



Any questions?

Q-Learning Update in Code

```
if np.random.rand(1) < epsilon:
   act = env.action_space.sample()
else:
   act = np.argmax(Q[st])
nst, rwd, done, _ = env.step(act)
Q[st][act] = (1-alpha)Q[st][act] + alpha(rwd + 
gamma*V[nst])
```

Q-Learning Code Demo

https://colab.research.google.com/drive/1yBCDzAXlu9j0A2aT8fH1zm7beBgtB9yY#scrollTo=sI_x2TsCp15L

Organizing RL problems/algorithms

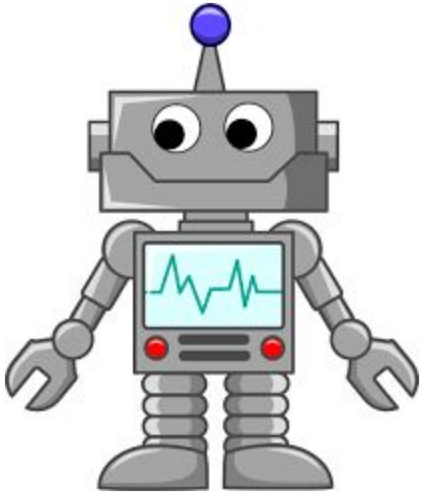
	Know T and R	Don't know T and R
Simple/discrete	Value iteration	Q-Learning
Complex/continuous	✕	**Deep Q-Networks** REINFORCE Actor-Critic

For a more complete taxonomy of RL algorithms, see https://spinningup.openai.com/en/latest/spinningup/rl_intro2.html#citations-below

Deep Q-learning

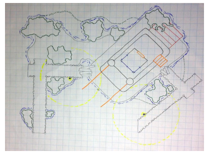
Limitations of Tabular Q-Learning

- Tabular methods can’t handle large or continuous state spaces
  - Can’t have a table for infinite Q or V values
- Lots of problems have these kinds of state spaces
  - Robotic navigation: The state of the robot could be (map of environment, position), where position is a 2-D vector. Infinite possible states in this setup.
  - **Go: has ~10^170 states**



https://en.wikipedia.org/wiki/File:Cartoon_Robot.svg

Map Position

s =  , $\binom{X}{Y}$

https://www.flickr.com/photos/mocklogic/8988098941

$argmax_a(Q(s,a))$

Action(Ex: Left, Right, Up, Down)

Too many possible values for a table

Beyond Tabular Learning

In Q-learning, we are learning a function Q that maps from a state-action pair to a real number

$$Q: (s, a) \rightarrow \mathbb{R}$$

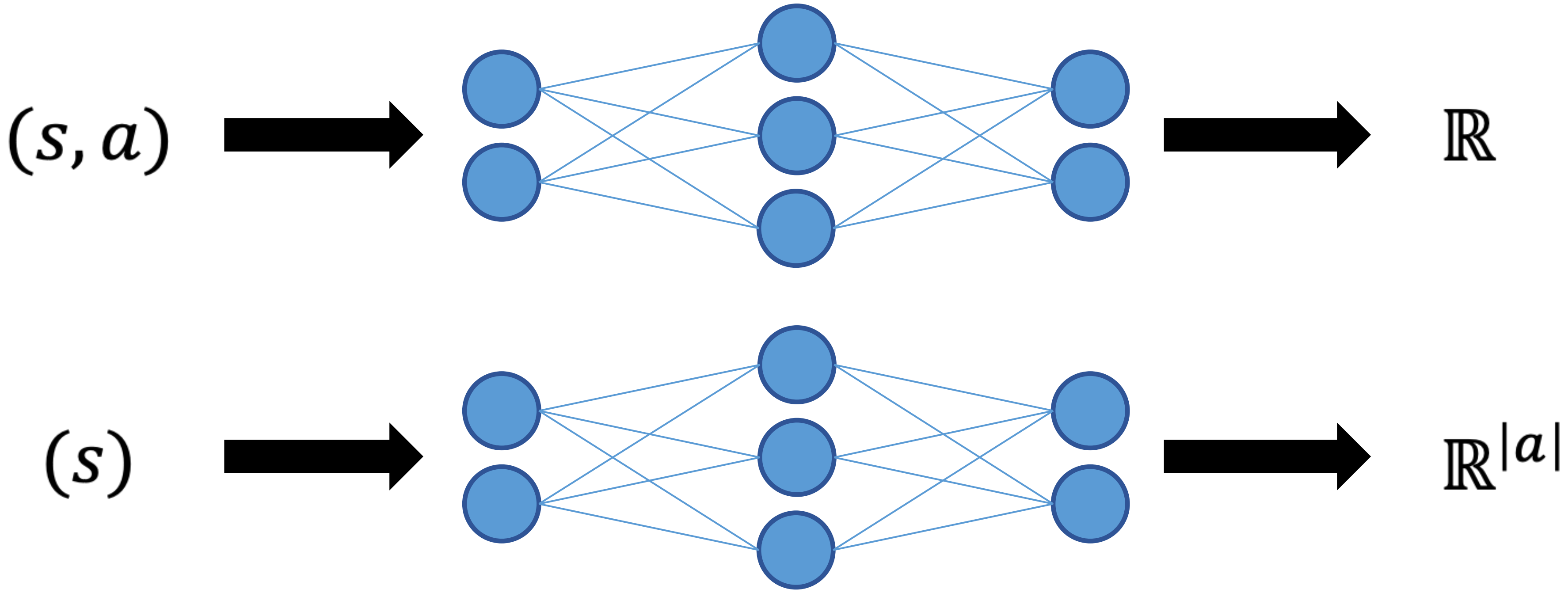
Or, equivalently, from a state to a vector of real numbers

$$Q: (s) \rightarrow \mathbb{R}^{|a|}$$

Instead of storing every (s,a) in a table to learn this function, we can learn a function to approximate Q using a (relatively) small set of parameters θ

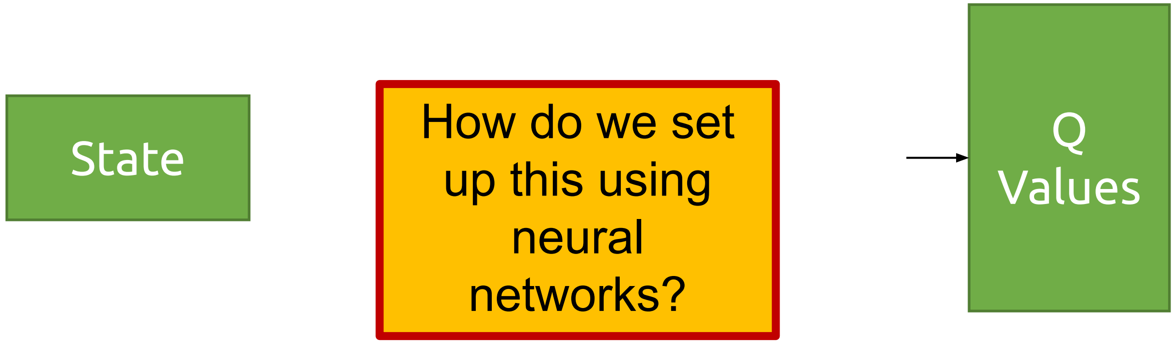
$$Q: (s, a, \theta) \rightarrow \mathbb{R}, \theta \ll |S \times A|$$

Neural Nets as Function Approximators



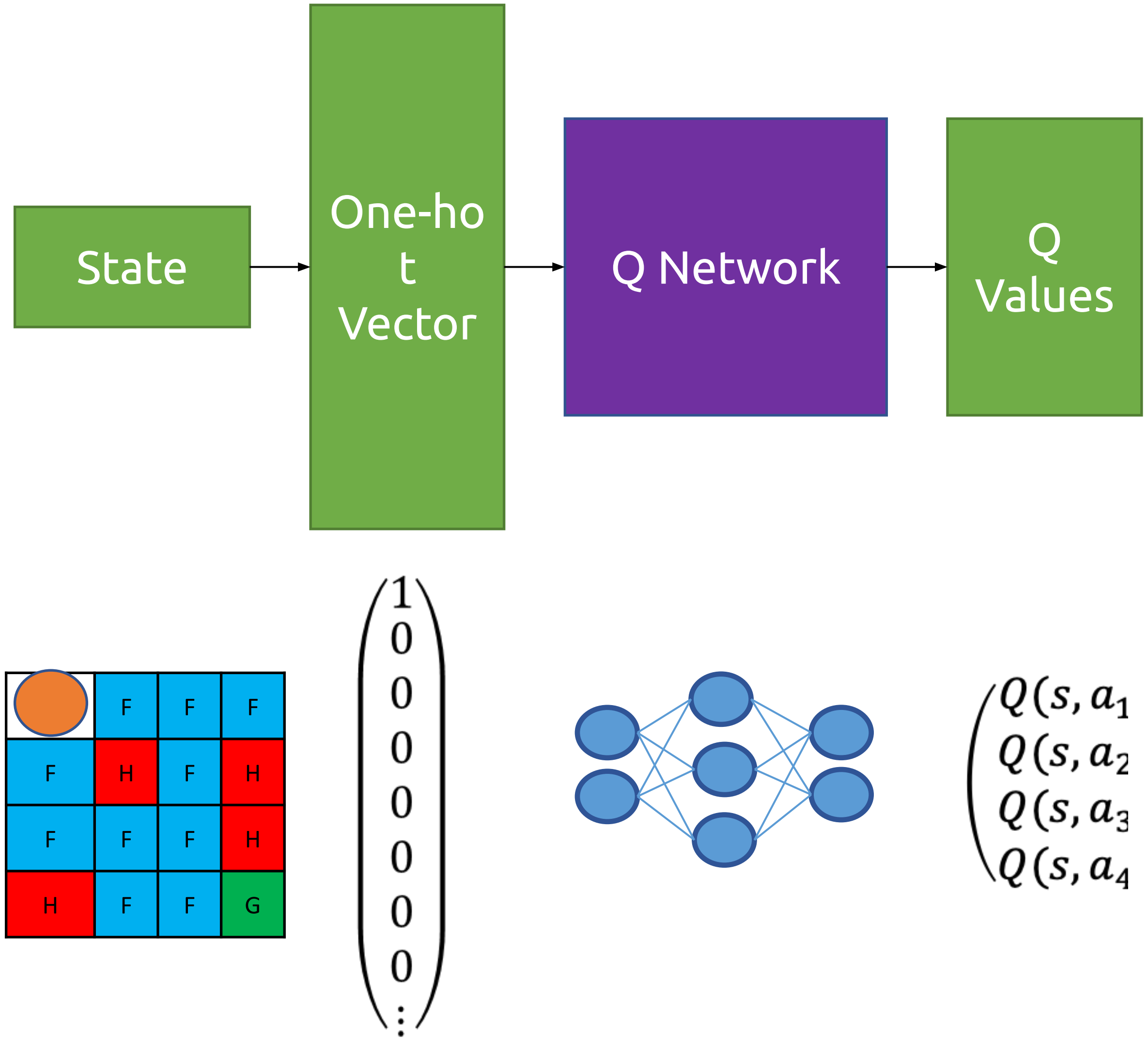
Neural networks are excellent function approximators, so we can use a deep network to learn this parameterized function

Frozen Lake Example



Frozen Lake Example

- Feed a one-hot representation of the state into a neural network to approximate the Q-value for each action
  - One-hot vector of length 16 for frozen lake
- Can also pass in a more complex state to the Q-network rather than a one hot representation.
  - Useful when the number of states is very large, making a one hot representation impractical
  - Or when states are continuous

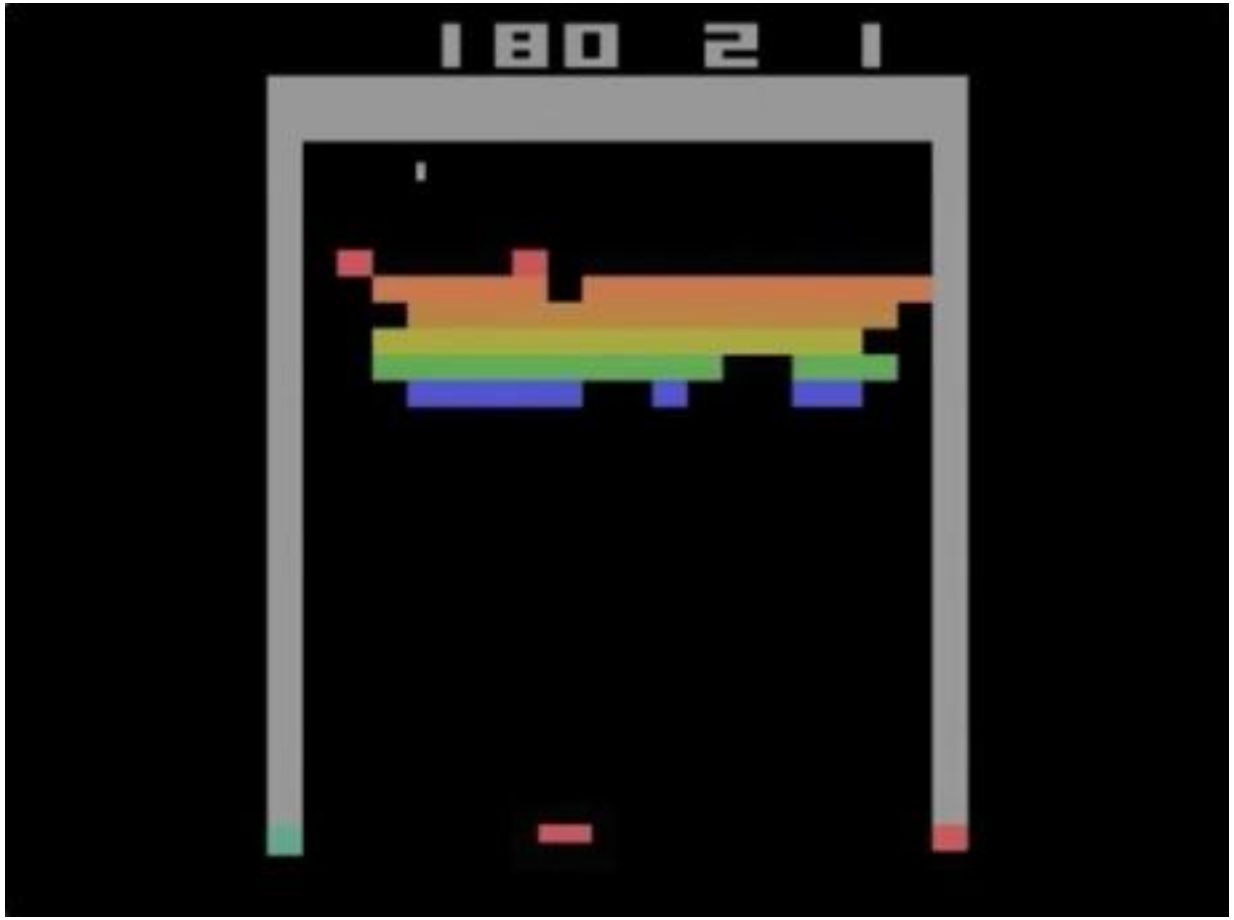


TF Example Code for Frozen Lake

```
# Weights for Q-Network
Q = tf.Variable(tf.random.uniform([16,4],0,0.01))
# Q-value function
def qVals(inptSt):
   oneH = tf.one_hot(inptSt,16)
   qVals = tf.matmul([oneH],Q)
   return qVals
# argmax over q-values to get estimated best action
action = tf.argmax(qVals(st),1)
```

Atari Example

- 81x81 Images of the game
  - Even if each pixel is just on or off, that's $2^{81\times81}$ possible states, way too many for a table
  - They're actually colored, so real scenario is even worse
  - Need a different approach to learn Q and V values for large state spaces like these



https://www.youtube.com/watch?v=TmPfTpjtdgg

How To Train This Q Network?

The original Q-learning update is not a minimization problem

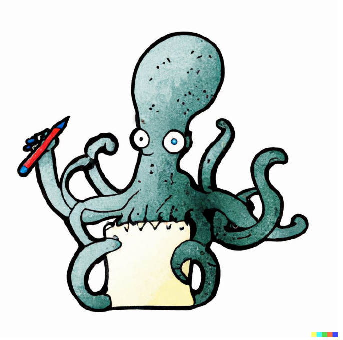
$$Q(s,a) = (1-\alpha)Q(s,a) + \alpha(R(s,a,s') + \gamma V(s'))$$

So how can we transform this into a loss function we can use?

Recap

Tabular Q-learning

- Wandering to estimate T and R
- Q-learning: Explore + Improve
- Epsilon-Greedy Policies



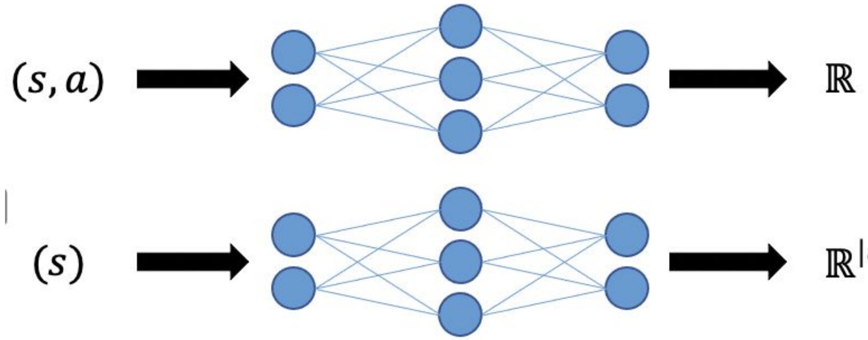
Deep Q-learning

- Limitations of Tabular Q-learning
- Neural nets for Q approximation
- Tensorflow code

S	F	F	F
F	H	F	H
F	F	F	H
H	F	F	G

$Q^{\pi}(s,a) = E[R(s,a,s') + \gamma V^{\pi}(s')]$

$Q^{\pi}(s,a) = \sum_{s' \in S} P(s'|s,a)[R(s,a,s') + \gamma V^{\pi}(s')]$



For more reading: https://huggingface.co/learn/deep-rl-course/unit2/introduction?fw=pt